Friction and adhesion in suspension rheology by constraint counting

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**In theory:** suspensions of hard non-Brownian particles should have a constant viscosity. **In practice:** they are highly non-Newtonian; there must be additional relevant stress scales in these systems.

**Rheology by constraints: revisiting shear thickening and friction**

Shear-thickening is driven by the onset of frictional contact at a critical stress, $\sigma^*$, when the stabilisation is overcome.

**Recasting Wyart & Cates theory**

Stress sets the proportion of frictional contacts, $f(\sigma)$: $f = \exp \left(-\left(\sigma/\sigma^*\right)^3\right)$

Schematic of jamming number, $Z(\sigma)$:

- $Z = 6 - 2f$
- $Z$ controls the jamming volume fraction $\phi_J(\sigma)$

**Introducing another constraint: adhesion**

Radial attraction does not add constraints; adhesion, resisting rolling, is the relevant effect. The strength will depend on both the interaction and on surface shape.

- It requires a contact area, which may arise from:
  - shape
  - deformability, or
  - roughness

The proportion of adhesive bonds $a$ will decrease from 1 to 0 with stress above $\sigma_A$; the strength of adhesion:

$$a(\sigma) = 1 - \exp \left(-\left(\sigma_A/\sigma^*\right)^3\right)$$

**References & Funding Acknowledgements**


**Conclusion:** many flow curves are captured by considering constraints (sliding, rolling, ...) at contacts. Rich behaviour arises from the balance of breaking adhesive bonds and making frictional contacts with changing stress.

I. **Weak adhesion:** $\sigma_A \ll \sigma^*$

The adhesive bonds are broken before frictional contacts are formed.

Both adhesion and friction introduce two constraints. A yield stress arises at the same $\phi$ as Discontinuous Shear Thickening (slope 1 on figure).

II. **Intermediate adhesion:** $\sigma_A \approx \sigma^*$

Here the flow depends sensitively on $\kappa/\beta$ and the balance of friction and adhesion.

Residual adhesion may explain the prolonged shear thinning seen in granular suspensions and their sensitivity.

III. **Strong adhesion:** $\sigma_A >\gg \sigma^*$

Attraction alone does not explain the yield stress of some granular suspensions.

The yield stress is instead driven by constraints and adhesion, depending strongly on inter-particle friction.

**Model flow curves, $\eta(\phi, \sigma)$**

**System:** 4\(\mu\)m sterically stabilised PMMA in CHB/decalin. $C = 3.35 \& \beta = 0.8$

**Use Krieger-Dougherty to calculate viscosity, $\eta(\phi, \sigma)$**

**System:** 45\(\mu\)m sterically stabilised PMMA [1]. $\beta = 1.0, \kappa = 0.4 \& C = 3.6$

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