1. Introduction
Shear-thinning fluids are characterised by a decreasing effective viscosity with increasing applied shear rate, $\gamma$. In pressure driven flows this leads to a plug like flow profile compared to the parabolic Newtonian profile, examples of these are plotted below.

An instability in pressure driven flow has been predicted theoretically in a number of models of shear thinning fluids [1-3]. Experimental evidence has recently been found to corroborate this [4-5].

2. Modelling Viscoelastic Behaviour
In order to study viscoelastic flows, constitutive models are used to describe the dynamics of the polymeric stress. To understand the behaviour of this instability more generally, we have used a number of models in this study. These have the form

$$\frac{D}{Dt}\Sigma=2\mathbf{G}+f(\Sigma\nabla\mathbf{v}) - \frac{1}{\eta}g(\Sigma), \quad \eta\nabla^2\mathbf{v} + \nabla \cdot \mathbf{v} - \nabla p = 0$$

Loading  Relaxation  Stokes Balance

where $\Sigma$ is the viscoelastic stress tensor, $\nabla\mathbf{v}$ is the rate of strain tensor, $G$ is the shear modulus. $D$ is the symmetric part of the rate of strain tensor such that $D=\frac{1}{2}(\nabla\mathbf{v}+(\nabla\mathbf{v})^T)$. Stokes balance and incompressibility, $\nabla \cdot \mathbf{v}=0$ are applied to the flow. Here $\eta$ is the solvent viscosity and $p$ is the pressure.

3. Linear Stability Analysis
We start by allowing the flow to evolve to a 1D base state, the velocity profile of which is shown in Figure 1. To this we add small amplitude perturbations with wavevector $q$ in the flow direction $x$ of the following form

$$\delta \Sigma_{ij}(x)e^{\omega t+iqx}, \quad \delta \psi(x)e^{\omega t+iqx}.$$ 

These perturbation then grow or decay according to the sign of $\omega$. Re($\omega$)>0 indicates the presence of an instability. Here $i$ and $j$ denote the indices of the viscoelastic stress tensor, $\Sigma$. $\psi$ is the stream function that describes the flow in the channel.

4. Linear Results
The eigenfunction associated with the instability tell us about the spatial structure of the disturbances. As seen in Figure 2, in the White-Metzner model the disturbances are rather delocalised across the channel, in the Johnson-Segalman model they are much more strongly peaked in the vicinity of a region of sharp variation in the underlying base state flow.

5. Non-linear simulations
Full nonlinear simulations explore the long time dynamics of the system. As can be seen, this resembles the finding of the linear calculation above, with much more strongly localised disturbances in the Johnson-Segalman model.

6. Conclusions
We observe a flow instability in two different models of shear thinning viscoelastic flow. Future work will explore the extent to which this instability is universal across shear thinning flows more generally.

References