Redistributive Innovation Policy, Inequality and Efficiency

Parantap Basu and Yoseph Getachew

Working Paper 2017 (02)
Redistributive Innovation Policy, Inequality and Efficiency

Parantap Basu\textsuperscript{a}, Yoseph Getachew\textsuperscript{a,b}

\textsuperscript{a}Durham University Business School, Durham University, Mill Hill Lane, DH1 3LB, Durham, UK
\textsuperscript{b}Department of Economics, University of Pretoria, 0028, Pretoria, South Africa

Abstract

Using a heterogenous-agent growth model with in-house R&D and incomplete capital markets, we examine the efficiency and distributional effects of alternative public R&D policies that target high-tech and low-tech sectors. We find that such policies have important implication for efficiency, inequality and social mobility. A regressive public R&D investment financed by income tax could boost growth and welfare via a positive effect on individual savings and effort. However, it could also discourage them via its effect on the efficiency–inequality trade off. The relationship between public R&D spending and welfare is therefore hump shaped admitting an optimal degree of regressivity in public R&D spending. A case for optimal progressive public R&D investment, however, can be made with a properly designed R&D policy that combines consumption tax and investment subsidy policies.

Key words:
Public R&D investment, inequality dynamics, social mobility, growth, welfare
JEL Classification: D31, E13, H4, O41

*Corresponding author
Email address: yoseph.getachew@up.ac.za (Yoseph Getachew)
1. Introduction

The role of public and private research and development (R&D) investment in economic growth is a widely debated topic.\(^1\) However, the distributional effect of public R&D investment has received little attention. In the extant literature, the focus is more on public education (e.g., Glomm and Ravikumar, 1992, 2003, Benabou, 2002) and infrastructure and taxes (e.g., Garcia and Turnovsky, 2007, Getachew, 2010, Chatterjee and Turnovsky, 2012, Getachew and Turnovsky, 2015). However, one can find ample evidence and intuition that public R&D policy could have uneven impacts on the economy. In many developing countries, innovation policy that focuses on the generation of drought resistant varieties and improved intercropping techniques might benefit marginal farmers more than proportional. On the contrary, R&D policy that focuses on the development of high yielding variety, fertilizers, large machinery and chemicals may tend to favour large commercialized farmers.

In general, most public R&D investment in developed countries are concentrated on high-tech industries such as information technology, biotechnology, communication, and environment industries. In the United States, for instance, public investment in equipment and software has increased from 20% in 1980 to 50% in 2001 (Cozzi and Impullitti, 2010).\(^2\) In contrast, in most of the developing world, a significant amount of public R&D investment are made in agriculture, a low-tech sector dominated by small scale farmers that constitute the majority of the population. Beintema et al. (2012) report that there is an accelerated public investment in agricultural R&D in developing countries during the period 2000 and 2008. The ratio of agricultural R&D spending to the global public spending increased from 39% to 46% in the developing world that includes middle and low income countries. The aim of

---

\(^1\) Particularly, in early 90s, there was an influx of R&D based growth theories, following the seminal works by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) that emphasize the role of R&D to economic growth, through influencing technological progress. R&D policies are also widely debated in terms of whether public R&D investment complements private R&D investment or crowds it out (e.g., David et al., 2000).

\(^2\) Kim et al. (2013) argue that R&D investment in South Korea is concentrated more on high-tech sectors.
this paper is to examine the efficiency, inequality and social mobility effects of such redistributive innovation policies.

A growing number of empirical studies try to assess the impact of different R&D policies on income inequality. Using provincial data in China for more than four decades, Zhang and Fan (2004) argue that government spending on agricultural R&D contributes to a reduction in regional inequality. Cozzi and Impullitti (2010) argue that government policy in R&D procurement plays a significant role in explaining the rising inequality in recent decades in the U.S.. They argue that an increase in public R&D investment in high-tech sectors of the economy in the early 1980s substantially boosted the relative wage of skilled workers. Our own calculations also suggest a contrasting relationship between inequality and public R&D spending, which is consistent with the previous studies. Figures 1 and 2 feature relationships between GINI index and the share of the public R&D spending in the U.S. and Sub-Saharan Africa (SSA), respectively.3 In both regions, public R&D spending has sharply increased during these periods, but the trend in inequality is quite opposite.4 While there is a positive correlation between these two variables for the U.S., the scatter plot for the SSA implies a negative correlation.5

---

3 The data come from World Bank (2015). Time average of GINI index and the share of public R&D are computed. Twenty two SSA countries, for which both inequality and R&D data are available are included. Inequality measure for each country is the time average of GINI from 1990 onward. Time series data for GINI are rather sparse for each country.

4 The average annual agricultural R&D spending growth in SSA countries, for instance, increased from 0.3% during 1981-1990 to 2.8% during 2000-2008 except for a small dip of .01% during 1990-2000, which is indicative of the bulk of recent R&D innovations in SSA being progressive in nature (Beintema et al., 2012).

5 The correlation coefficients between GINI and R&D spending ratio are 0.81 and -0.25, significant at 5% level for the U.S. and SSA, respectively. Dropping South Africa as an outlier (which has the highest GINI coefficient of 0.62) raises the correlation coefficient to -0.41, which is also significant at 5% level. For the U.S., the high correlation could be due to a common trend as both variables show a secular increase. Taking out the trend using a Hodrick–Prescott (HP) filter changes the correlation to 0.35.
This paper develops a heterogenous-agent growth model with in-house R&D where both inequality and growth are endogenously determined. In the model, agents are heterogenous in terms of their initial endowments of knowledge and their ability to generate knowledge. The source of endogenous growth (technological progress) is in-house R&D investment using private and public resources. Endogenous inequality is generated due to missing credit and insurance markets, as in Loury (1981) and

Using this framework, we analyze the effects of progressive and regressive public R&D investment policies on inequality, social mobility, growth and welfare. For each policy, we ask what degree of regressivity/progressivity is efficient, or growth- and welfare-maximizing. We find that regressive (progressive) R&D investment policy essentially aggravates (mitigates) inequality and slows down (speeds up) social mobility. However, the relationship between efficiency and redistributive innovation policy is nonlinear. Regressive innovation policies have the benefit of promoting efficiency, due to incentives on agents’ savings and effort decisions. Progressive policies also have benefits, due to imperfections in capital markets; redistributive R&D policy then provides a means to relax the credit constraints that impede certain investments. When calibrating the model for reasonable parameter values, the relationship between public R&D spending and welfare is hump shaped and admits an optimal degree of regressivity in public R&D spending. However, a novel case for optimal progressive R&D policy can be provided through a properly designed tax-subsidy policy. In particular, a consumption tax and investment subsidy can be designed to correct some of the negative effects on private investment and restore it to a regime with neutral R&D policy.

The paper connects to a wider literature in inequality, social mobility and growth. It relates to the literature that analyzes the growth–inequality trade off under imperfect credit markets (see, for e.g., Loury, 1981, Galor and Zeira, 1993, Aghion and Bolton, 1997, Aghion et al., 1999, Benabou, 2000, 2002, 2005 and Basu and Getachew, 2015). Our focus on redistributive productive public goods also connects us to the literature in public education and inequality, and, infrastructure and inequality, despite the scant attention they pay to public R&D investment. The

---

6This literature mainly abstracts from productive public spending issues, however. For instance, Benabou (2005) focuses on the distributional and growth impact of progressive taxation.
7See, for example, Glomm and Ravikumar (1992, 2003), Saint-Paul and Verdier (1993) and Eckstein and Zilcha (1994) among many other.
8See, for e.g., Garcia-Penalosa and Turnovsky (2007), Getachew (2010, 2012), Chatterjee and
paper mainly complements, but differ, to the growing literature on innovation and inequality (e.g., Chu, 2010, Cozzi and Impullitti, 2010, and Aghion et al., 2015). This literature focuses on the effects of innovation or innovation policy on inequality and social mobility.\(^9\) We provide a novel mechanism related to the optimality of alternative redistributive innovation policies.\(^{10}\)

The paper is organized as follows: The next Section develops the model. Section 3 characterizes individual and aggregate (inequality) dynamics. Section 4 discusses the effects of different R&D policies on steady-state growth and inequality while Section 5 focuses on optimality of alternative R&D policies. Section 6 concludes.

2. The Model

We assume that the economy is populated with a continuum of heterogenous agents, \(i \in (0, 1)\). There is no population growth in the economy. The first generation of the \(i\)th agent is endowed with \(h_{i0}\) levels of knowledge. Initial distribution is given and assumed to take log-normal, \(\ln h_{i0} \sim N(\mu_0, \sigma_0^2)\), which evolves endogenously at equilibrium. Agents also differ in their respective productivity and creativity to generate income and knowledge, respectively, where both are assumed to be i.i.d. and log-normally distributed. Combined with labour, knowledge is used to produce intermediate goods, which are, in turn, used for production of the final goods.

There are three sectors in the economy, namely the final goods, the intermediate goods and the knowledge production sectors. Using a CES production function, a competitive firm transforms intermediate inputs into a final good. These differentiated intermediate inputs are produced by competitive firms using a convex technology. Each firm in this sector invests in an in-house R&D, in the spirit of Smulders

---

\(^9\)Chu (2010), for instance, argues that strengthening patent policy increases income inequality by raising the return on assets. Whereas, Aghion et al. (2015) focus on the relationship between innovation, top income inequality and social mobility for the U.S.

\(^{10}\)Our paper may also relate to the strand of literature that attributes the recent rise in inequality in many advanced economies to skill biased technical change (see, for instance, Acemoglu, 2002 and Aghion, 2002) although this literature often focuses on private R&D investment or technical progress and abstracts from optimal public R&D policy.
and Van de Klundert (1995), to expand a specialized know-how that is required to produce a specialized input. The production of knowledge requires both the use of public and private resources, and a backlog of knowledge stock. The government levies a fixed flat rate tax on the income of individual agents to finance the ‘public good’. This public good is provided disproportionately among rich and poor agents to supplement private R&D investment.

2.1. Final goods

In the spirit of Benabou (1996), the final goods and services are produced using a continuum of differentiated intermediate inputs \(x_{it}\) by the representative firm in the economy:

\[
y_t = a_1 \left( \int_0^1 \phi_{it} x_{it}^{(\varepsilon-1)/\varepsilon} \, di \right)^{\varepsilon/(\varepsilon-1)} ; \varepsilon > 1
\]

where \(x_{it}\) is the intermediate input supplied by the \(i\)th intermediate goods firm and \(a_1\) is a deterministic TFP parameter; \(\phi_{it}\) represents idiosyncratic productivity shocks, which are i.i.d. with mean one and a constant non-zero variance, attached to each intermediate input:

\[
\ln \phi_{it} \sim N \left( -\kappa^2/2, \kappa^2 \right)
\]

\(\varepsilon > 1\) is the elasticity of substitution between the intermediate inputs, which determines the firms’ monopoly power, in the spirit of Dixit and Stiglitz (1977).

Profit maximization by the perfectly competitive firm, given a unit price of the final goods, leads to the downward slopping input demand function:

\[
x_{it} = \phi_{it} a_1^{1-\varepsilon} y_t \left( \frac{1}{p_{it}} \right)^{\varepsilon}
\]

where \(p_{it}\) denotes the price of the \(i\)th intermediate good and \(-\varepsilon\) is the price elasticity of demand.

2.2. Intermediate goods

The differentiated intermediate goods firms are characterized with certain features. First is the presence of specialization. Knowledge is firm-specific, and hence
the production of intermediate goods. Thus, each intermediate goods firm has some monopoly power over its price. Consequently, the rate of returns and earnings are different among firms in this sector. Second, a firm in this sector engages in an in-house R&D investment to expand its specialized knowledge stock. The R&D investment is the only vehicle of technical progress.

As in Aghion et al. (2015), the $i$th firm in the intermediate goods sector needs $1/h_{it}$ units of labour to produce one unit of its variety:

$$x_{it} = h_{it}l_{it}$$

(3)

where $h_{it}$ represents the stock of the firm specific knowledge, generated through in-house R&D activity, which is specified below; and, $l_{it}$ is the raw labour input. Each period, the firm’s profit consists of revenue from the sale of the intermediate good, $x_{it}$, net of the total labor cost ($l_{it}w_{it}$) where $w_{it}$ is the wage rate per unit of labor. Thus, the firm has the following static optimization problem,

$$\max_{\{l_{it}, p_{it}\}} \pi_{it} = p_{it}(x_{it},.)x_{it} - w_{it}l_{it}$$

subject to the demand function (2). The first order condition leads to the following pricing:

$$p_{it} = (w_{it}/h_{it})\varepsilon/(\varepsilon - 1)$$

(4)

While $w_{it}/h_{it}$ is the marginal cost of producing a unit of the intermediate input, the elasticity of substitution, $\varepsilon$, determines the mark-up over this cost.

The $i$th agent income is given by $y_{it} = l_{it}w_{it}$. Substituting (2), (3) and (4) into this, one obtains:

$$y_{it} = a\phi_{it}(l_{it}h_{it})^{\alpha}y_{i}^{1-\alpha}$$

(5)

where $a \equiv \alpha^{-1}a_{it}^{\alpha}$ and $\alpha \equiv (\varepsilon - 1)/\varepsilon$.

Eq. (5) matches individual income to output production, characterized by constant returns to scale at individual ($h_{it}$) and aggregate accumulative factors ($h_{i}$) in
However, there is diminishing returns to individual factor. This shows that the model is basically in the spirit of the Arrow (1962) and Romer (1986) learning-by-doing endogenous growth models.

2.3. In-house R&D

Each intermediate goods firm invests in an in-house R&D to produce the know-how using the following knowledge production function:

$$h_{it+1} = \zeta_{it+1} h_t^\theta s_t^\nu g_t^\lambda$$

(6)

Government intervenes in the R&D process by investing in public R&D input \((g_{it})\) that uses to complement the private sector, but with a redistributive intent. According to (6), knowledge is a product of both public and private investment \((g_{it} \text{ and } s_{it}, \text{ respectively})\), past knowledge stock of the firm \((h_{it})\) and idiosyncratic shocks \((\zeta_{it+1})\). \(\{\theta, \lambda, \nu\} \in (0, 1)\) denote knowledge elasticities. \(\zeta_{it+1}\) is i.i.d. and follows a log-normal distribution with mean one and a constant variance,

$$\ln \zeta_{it+1} \sim N \left(-\sigma^2 / 2, \sigma^2 \right)$$

The production function (6) exhibits constant-returns to scale \(\theta + \nu + \lambda = 1\), which makes the growth process endogenous as in any standard growth model.

2.4. Government budget

Public R&D investment is financed using a proportional income tax \((\tau)\), which is levied in the final goods. The government balances the budget as in the growth and public investment literature (e.g., Barro, 1990):

$$g_t = \tau \int_0^1 y_{it} di = \tau y_t$$

(7)

where \(g_t\) denotes the total public investment in R&D and \(\tau\) is the public expenditure GDP ratio. Thus, a fraction of aggregate income is used to finance the public good.

---

\(^{11}\text{As we see later, } y_t \text{ is a linear function of } h_t \text{ and } l_{it} = l, \text{ which is constant.}\)
The key feature of this paper lies in the relationship between the production of knowledge and public expenditure. We abstract from a blanket public investment provision in R&D. Rather the government expenditure in R&D has a redistributive component. Public R&D investment does not necessarily benefit individual firms proportionally. Small firms may benefit disproportionately from low-tech technologies as large firms do from high-tech technologies. For instance, an innovation of a pedal-powered tractor is more beneficial to small-scale farmers, as a high-powered tractors for large commercial farms. Formally, this can be expressed as

$$g_{it} = (h_{it}/h_t)\omega g_t$$

The key redistributive R&D policy parameter is $\omega$. Its magnitude and sign determine the weight and nature of redistribution, respectively. If $\omega = 0$, for instance, $g_{it} = g_t$ is a pure public good where all firms equiproportionately benefit from public R&D. We call such a R&D policy neutral because it is not biased towards the rich or poor. A positive $\omega$ implies merit-based public expenditure. R&D firms with a relatively high level of initial knowledge stock compared to the average knowledge stock (meaning a higher $h_{it}/h_t$) benefit more than proportional from public R&D spending ($g_t$). A negative $\omega$, on the other hand, makes small firms with a relatively lower level of initial knowledge stock (a lower $h_{it}/h_t$) benefit more from public spending on R&D. Hereafter, we refer to negative $\omega$ and positive $\omega$ as progressive and regressive public expenditure, respectively, in line to the literature in progressive/regressive taxation.\(^\text{12}\)

Combining (6) and (8) one obtains:

$$h_{it+1} = a_2\zeta_{it+1} h_{it}^{\theta+\omega\lambda} s_{it'}^\nu (g_t/h_t^\omega)^{\lambda}$$

\(^\text{12}\)In Benabou (2000, 2002), for instance, after-tax income is given by $\tilde{y}_{it} = (y_{it}/\bar{y}_t)^\tau \bar{y}_t$ where $y_{it}$ and $\bar{y}_t$ represent before-tax and threshold incomes, respectively; $\tau$, which has basically a similar role as $\omega$ has in our model, represents the marginal tax rate whose sign determines the progressivity/regressivity of the tax schedule. We differ from this literature, however, as we focus on the expenditure side.
The parameters $\theta$ and $\lambda$ are *ex ante* knowledge elasticities whereas $\theta + \omega \lambda$ and $\lambda - \omega \lambda$ capture *ex post* intergenerational linkages associated with firm level knowledge production that account for individual and aggregate factors in the economy, respectively.\(^{13}\) The term $\omega \lambda$ captures the redistributive nature of the public variable and its implication for individual knowledge accumulation. Redistribution thus impacts the economy via the effect on private and public knowledge elasticities. We see later individual optimal decision is crucially dependent on $\theta + \omega \lambda$, which is also the main determinant of the evolution of inequality, which, in turn, determines other macroeconomic dynamics.

2.5. Household

There is a continuum of households indexed between $(0, 1)$. Households own the firms.\(^{14}\) Similar to Benabou (2002, 2005), the credit and insurance markets are missing. We also assume members of the households are endowed with units of labour that they supply elastically. Agents maximize their utility in accordance of the following function:

$$U_{it} \equiv \max_{\{c_{it}, h_{it+1}, l_{it}\}_0} E_t \sum_{t=0}^{\infty} \rho^t (\ln c_{it} - l_{it}^\eta)$$

where $\eta > 1$; $E_t$ is an individual's expectation given information at date $t$. The budget constraint is given by:

$$c_{it} + s_{it} = (1 - \tau) y_{it}$$

where $\tau$ represents income tax.

Applying standard methods, individual household decision rules can be derived as follows:

\(^{13}\)As we shall soon see $g_t$ is a linear function of $h_t$.

\(^{14}\)Other models that use similar type of individual entrepreneurship include Benabou (2000, 2002, 2005) and Angeletos and Calvet (2005, 2006).
\[ s_{it} = b (1 - \tau) y_{it} \quad (12a) \]
\[ l_{it} = l = (\alpha / (\eta (1 - b)))^{1/\eta} \quad (12b) \]
\[ c_{it} = (1 - \tau) (1 - b) y_{it} \quad (12c) \]

where

\[ b \equiv \rho \alpha \nu / (1 - \rho (\theta + \omega \lambda)) \]

Eqs. (12) are standard forms from the viewpoint of household optimization. Households supply a constant unit of labour while saving rate is independent of rate of returns, as a consequence of log utility function. Both saving rate and effort increase with the discount factor (\( \rho \)), elasticity of substitution (\( \varepsilon \)), intergenerational spillover (\( \theta \)), and the elasticity of private investment (\( \nu \)). But, the effect of \( \omega \) and \( \lambda \) on saving rate and labor supply depend on the sign of \( \omega \). Both increase if the R&D program is regressive (\( \omega > 0 \)) while they decrease if it is progressive (\( \omega < 0 \)), the classic efficiency–equity trade off. We thus have the following proposition:

**Proposition 1.** Saving rate and labour supply increase (decrease) with regressive (progressive) public R&D investment.

**Proof.** See eqs. (12a) and (12b). ■

2.5.1. Aggregate consumption, investment and income:

Aggregate consumption and savings are given by

\[ c_t = (1 - \tau) (1 - b) y_t \quad (13) \]
\[ s_t = (1 - \tau) by_t \quad (14) \]

Aggregate income is derived from aggregating (5), after substituting (12b):

\[ y_t = la^{1/\alpha} h_t \exp (d_t) \quad (15) \]
where $d_t$ is a composite parameter, which captures the relationship between aggregate income and inequality:

$$d_t \equiv 0.5(\alpha - 1)\sigma_t^2$$

In this case, the government budget constraint is given by, from (7) and (15):

$$g_t = \tau l a^{1/\alpha}h_t \exp(d_t)$$

(16)

where $l$ is given by (12b). Given that individuals’ income is determined by their optimal labor supply, aggregate labor is an important component of aggregate income and hence aggregate public R&D expenditure. In addition, considering $d_t < 0$, the existence of diminishing returns in individual income combined with missing credit markets implies that aggregate income, and hence public R&D decrease in inequality.

3. Dynamics

3.0.2. Optimal knowledge dynamics and intergenerational mobility

The dynamics of optimal knowledge stock associated to the $i$th firm is derived from (5), (9) (12a), (12b), (15) and (16):

$$h_{it+1} = a_3\psi \zeta_{it+1}^\nu \phi_{it}^\beta h_{it}^\kappa \exp((\lambda + (1 - \alpha)\nu) d_t)$$

(17)

where $a_3 \equiv (\alpha/\eta)^{(\nu+\lambda)/\eta} a^{(\nu+\lambda)/\alpha}$ and

$$\psi \equiv b^\nu (1 - b)^{-\nu - (\nu + \lambda)/\eta}$$

$$\chi \equiv \tau^\lambda (1 - \tau)^\nu$$

$$\beta \equiv \theta + \omega \lambda + \alpha \nu$$

$$\kappa \equiv \lambda + (1 - \alpha) \nu - \omega \lambda$$

Eq. (17) captures the optimal dynamics of knowledge at a firm level. $\tau$ and $\omega$ are policy parameters while the rest are structural parameters. The government adjusts the size of investment through its choice of $\tau$ whereas the sign of $\omega$ determines
the redistributive nature of the public fund. Such policy variables impact the TFP of individual knowledge production function via their effects on individual savings, effort and public R&D investment. These are in particular reflected in $\psi$ and $\chi$. As captured in $\chi$, there is a positive effect from income tax through its financing of public R&D expenditure; but, a negative effect in its distortionary effects on individual savings. The resultant effect is determined by the weight of the respective elasticities. From $\psi$, redistribution ($\omega$) affects individual knowledge production through its effects on individual saving rate and effort. Redistributions also impact the elasticities of past knowledge ($\beta$, $\kappa$) with an important implication to inequality dynamics.

The dynamics of optimal individual knowledge also depends on the current individual and aggregate knowledge variables, idiosyncratic risks both in the final goods ($\phi_{it}$) and R&D sectors ($\zeta_{it+1}$) and current inequality. Risks in the final goods sectors affect individual savings and investment indirectly via individual income whereas those in the knowledge sector have a direct impact. The exponential term in (17) captures the relationship between inequality and individual knowledge dynamics. $d_t < 0$ reflects the negative effects of inequality on knowledge production at firm level. Through its impacts on aggregate and subsequent individual savings and investment, inequality negatively impacts individual knowledge accumulation.

3.1. Intergenerational mobility

The model has a direct implication for intergenerational mobility, as in Basu and Getachew (2015). The intergenerational elasticity (IGE) of knowledge ($\beta$) is derived from (17), first by taking logs from both sides of the equation, and then computing the partial derivative of the next-period knowledge with respect to the current knowledge:

$$\beta \equiv \frac{\partial \ln h_{it+1}}{\partial \ln h_{it}} = \theta + \omega \lambda + \nu \alpha$$

(19)

$1 - \beta$ is a measure of intergenerational mobility.

We are measuring mobility in terms of knowledge while in majority of the literature it is measured in terms of income (see, for instance, Solon, 1992 and Mazumder, 2005, among others). Given that knowledge is the only input in the production
function, it is straightforward to verify from (12a) that $\beta$ also governs the intergenerational persistence of income.

According to (19), intergenerational mobility is independent of the idiosyncratic shocks, but it depends crucially on the structure of goods and knowledge production and knowledge accumulation technologies at the individual household level.

**Proposition 2.** *Intergenerational mobility increases in progressive public R&D expenditure ($\omega < 0$), and conversely.*

**Proof.** See (19). ■

3.2. Inequality dynamics

The dynamics of inequality is also derived from (17), by taking the log and variance,

$$\sigma_{t+1}^2 = \nu^2 \sigma_t^2 + \sigma_t^2$$

Given $\beta \in (0, 1)$, (20) is a stable dynamics that converges to a steady state inequality. The variance of the idiosyncratic shocks ($\sigma_t^2$ and $\sigma_t^2$) will determine the long-run property of the model. Volatility in the final goods sector affects inequality via its effect on individual savings while volatility in the R&D sector directly impacts inequality dynamics. The root of the dynamics of inequality is determined by $\beta$, which, in turn, is a function of policy and structural parameters, $\varepsilon$, $\lambda$, $\omega$, $\nu$ and $\theta$. Higher intergenerational linkage (higher $\theta$) results in higher transitional inequality. Better private investment technology (higher $\nu$) implies slower convergence in inequality. Private R&D investment elasticity ($\nu$) also impacts inequality through individual response to luck, with a strong implication to long-run inequality.

The effect of the public variables on the dynamics of inequality rather depends on its redistributive feature (the sign of $\omega$). If $\omega < 0$, higher elasticity of public R&D investment (higher $\lambda$) leads to faster convergence of inequality, and conversely. If $\omega = 0$, i.e. public investment in R&D is proportionally provided, however, the elasticity $\lambda$ has a neutral effect in inequality. We thus have the following proposition:
Proposition 3. A regressive (progressive) R&D investment aggravates (mitigates) transitory inequality. In other words, if $\omega > 0$ ($\omega < 0$), given $\sigma_t^2$, $\sigma_{t+1}^2$ increases (decreases) in $\omega$, and conversely.

Proof. From (20), if $\omega < 0$, for given $\sigma_t^2$ then $\sigma_{t+1}^2$ decreases in $|\omega|$, and conversely.

Note also that slower mobility (higher $\beta$) also implies slow convergence of the inequality dynamics. That is, the greater $\beta$ is, the more persistent inequality becomes. However, since the coefficient in the inequality dynamics ($\beta^2$) is smaller than the mobility coefficient ($\beta$), intergenerational immobility is much more persistent than inequality.

4. Steady state

Note that given $\beta \in (0, 1)$, which is the sufficient condition for the stability of the distributional dynamics, (20) converges to a unique inequality equilibrium. But, with constant-returns to scale in knowledge production, inequality is the only source of dynamics in the economy. As inequality converges to its equilibrium level, growth also converges to its steady-state level. In this case, long-run inequality and growth are given by, from (20) and aggregating (17), respectively:

\[
\begin{align*}
\sigma^2 &= \frac{(\nu^2 \kappa^2 + \varphi^2)}{(1 - \beta^2)} \\
1 + \gamma &= a_2 \psi \chi \exp\left(\sigma^2 (\theta + F + q)\right)
\end{align*}
\]

where

\footnote{Appendix A provides a detail on the derivation of the balanced growth path.}
\[ q \equiv 0.5 (\alpha - 1) (\lambda + (1 - \alpha) \nu) < 0 \]  \hfill (22a)  
\[ F \equiv 0.5 \beta (\beta - 1) < 0 \]  \hfill (22b)  
\[ \pi \equiv 0.5 \nu (\nu - 1) z^2 < 0 \]  \hfill (22c)  

\( \gamma \) is the steady-state growth rate of the economy. The long-run equilibrium of the economy is a balanced growth path, with a constant non-zero level of inequality (see Appendix A).

A number of interesting results could be drawn from (21): Firstly, steady-state inequality increases in IGE and volatility. Secondly, Proposition 3 also holds in the steady state. Thirdly, inequality has a negative impact on long-run growth. This is easily seen as all the terms in the bracket in (21b) are negative. Finally, redistributive policy (sign of \( \omega \)) impacts long-run growth directly, via its effect on agents’ savings and effort, and indirectly, via the growth–inequality trade off.

5. Optimal redistributive policies

5.1. Growth maximizing policies

Should the government innovation policy be progressive or regressive to maximize growth?\(^\text{16}\) For a homogeneous economy (especially, \( h_{i0} = h_0 \)), the choice of \( \omega \) is straightforward. In order to maximize growth, public policy should be regressive (\( \omega > 0 \)) and take the maximum attainable value. Because, in this case regressivity has only the benefit of promoting efficiency, due to incentives on agents’ savings and effort decisions.

For the heterogenous case, however, this may not be necessarily true. The policy effect on growth could be rather nonlinear, as both inequality (\( \sigma^2 \)) and \( \psi \) could

\(^\text{16}\) Although our main focus here is on \( \omega \), note that both \( \omega \) and \( \tau \) are policy variables. With respect to \( \tau \), the growth maximizing tax rate (\( \tau^* \)) could easily be computed from (21b): \( \partial \gamma / \partial \tau \equiv \tau^* = \lambda / (\lambda + \nu) \), which is independent of redistribution, \( \omega \). \( \tau^* \) reaches its upper bound when \( \nu = 0 \) – when there is, no, or, little private investment in R&D.
increase in $\omega$. On the one hand, a regressive R&D policy encourages growth due to its positive impact on savings and effort; on the other, it has a negative impact on growth via the inequality–growth trade off.

**Proposition 4.** (i) If $\sigma^2_0 = 0$, a regressive R&D policy unambiguously promotes growth. (ii) If $\sigma^2_0 \neq 0$ and $\varkappa^2 \neq 0$ (or $\vartheta^2 \neq 0$), the long-run growth effect of $\omega$ is ambiguous.

**Proof.** See Appendix C. ■

We can have further insight on the nonlinearity, if we specify parameter values that seem reasonable for real economies. We thus set the subjective discount factor $\rho$ at 0.99, as in numerous macroeconomic studies. Following Benabou (2002), we set the intertemporal elasticity of substitution $\epsilon \equiv 1/(\eta - 1)$ to 0.20. The elasticity of substitution between intermediate goods is fixed at 6 (Kollman, 2002). The initial knowledge ($h_0$) and the initial distribution of knowledge ($\sigma^2_0$) are normalized at unity. Getachew and Turnovsky (2015) consider a standard deviation for the logarithm of idiosyncratic shocks. Based on that we set $\chi^2 = \varrho^2 = 0.16$.

Using the World Bank (2015) database for the period 2005-2014, the average public and private R&D spending GDP ratio is computed as 2.81% for the U.S. The public R&D spending ratio ($\tau$) is thus 0.548%. The parameter $\nu$ represents the elasticity of knowledge production with respect to private R&D spending. Following Jones and Williams (2000), we set $\nu = 0.5$. The value for the elasticity of public knowledge widely varies among empirical estimates. We use Levy’s (1990) estimate for the public investment elasticity of private R&D for nine OECD countries between 1963 and 1984, which is about 0.34 and consistent to the estimate’s of Leyden and Link (1991). This implies $\theta = 0.16$. Finally, we calibrate $a_1$ to reproduce a 2% average annual growth rates of GDP.

Applying these values to (21b), we find that the growth maximizing policy is quite regressive (see Fig. 4). Growth is in particular maximized when $\omega$ is slightly

---

17 See David et al. (2000) for the survey of the literature.
18 Leyden and Link’s estimate is based on a 1987 data set of 137 R&D laboratories for the U.S. industries.
greater than unity, $\omega = 1.09$.\footnote{Note that there is a restriction on $\omega$ for a stable inequality dynamics (20), $0 < \beta < 1$. Given our calibrated values, this means $-1.6961 < \omega < 1.2451$.}

Fig. 4: Growth Maximizing Redistributive R&D Policy

5.2. Welfare maximizing policies

While growth and inequality are important macroeconomic variables, the economic significance of any policy should be basically judged in terms of its impacts on social welfare. Given that $V_{i0} = \max U_{i0}$ is the discounted sum of individual welfare, its aggregation across the entire population leads to the discounted sum of aggregate welfare: $W_0 = \int_0^1 V_{i0} di$. Then, the steady-state aggregate welfare is given by (see Appendix B for details)

$$W = (1 - \rho)^{-1} \left( \ln c_0 - 0.5 \sigma^2 c - \ln \theta \right) + \rho / (1 - \rho)^2 \ln (1 + \gamma)$$

(23)
where
\[
c_0 = (1 - \tau)(1 - b) y_0 = (1 - \tau)(1 - b) l a^{1/\alpha} h_0 \exp(0.5(\alpha - 1) \sigma_0^2) \tag{24}
\]
\[
\sigma_c^2 = \chi^2 + \alpha^2 \sigma^2
\tag{25}
\]
where $c_0$ and $\sigma_c^2$ are initial aggregate consumption and consumption inequality, respectively.

The first term in (23) captures the discounted initial aggregate welfare (at $t = 0$). Given that $h_0$ is predetermined, so is $\sigma_0^2$, which has a negative impact on welfare. Since individuals derive a negative utility from increased effort, the policy effects ($\omega$) on the initial welfare is nonlinear. While a higher $\omega$ may negatively affect initial welfare via an effect on efforts and inequality, it may increase it through boosting initial income (higher $y_0$).

The second term in (23) comes from the economy-wide growth rate, which captures individuals' rewards for saving and investing in their future. In this case, any effect policy has on growth would directly pass to welfare. A higher $\omega$ unambiguously raises the steady state inequality, $\sigma^2$ as seen from (21a) and through this channel it lowers growth and hence welfare. On the other hand, it promotes investment and raises effort that could in turn raise growth and thus welfare.

Should the government R&D policy be progressive or regressive to maximize welfare? For reasonable parameter values, we see that welfare maximizing policy is also regressive due to a strong growth effects on aggregate welfare (see Fig. 5). Aggregate welfare is in particular maximized when $\omega = 1.07$, which is slightly less regressive than the case for growth due to an additional adverse effect of a regressive policy on initial welfare.
5.3. Is there a case for optimal progressive R&D policy?

As seen in Proposition 1, a negative $\omega$ depresses individual savings and effort. Thus, it adversely affects growth via its negative effects on private investment and labour supply although it has a positive effect on inequality and hence growth. However, a consumption tax and investment subsidy can be designed to correct some of these negative effects on private investment and restore it to a regime with neutral R&D policy.\footnote{It may be possible to restore the distortion in savings, but not in effort, due to a one-to-one correspondence in consumption and investment goods.}

Let the government subsidizes individual savings (at a rate of $\vartheta$) using a consumption tax. In this case, (6) becomes:

$$h_{it+1} = a_2 z_{it+1} h^\vartheta_{it} (1 + \vartheta) s_{it}^{\nu} g_{lt}^\lambda$$  \hspace{1cm} (26)

where $\vartheta$ is the subsidy rate. Therefore, the individual receives an additional amount
of \( \vartheta s_{it} \) subsidy for \( s_{it} \) level of investment. If the government chooses to finance this with a consumption tax at a rate of \( \tau_c \), then individual and (the balanced) government budget constraints become respectively,

\[
(1 + \tau_c) c_{it} + s_{it} = (1 - \tau) y_{it}
\]

and

\[
\vartheta s_{it} = \tau_c c_{it}
\]

The rest of the government budget is given separately by (7). With consumption tax, only individual optimal consumption will be affected.

\[
c_{it} = (1 - b) (1 - \tau) / (1 + \tau_c) y_{it}
\]

Since all individuals face the same consumption tax rate \( \tau_c \) and investment subsidy rate \( \vartheta \), the optimal individual saving rate and the labour supply remain the same as (12a) and (12b) respectively as in the case of no tax-subsidy. The optimal solution for consumption, however, decreases by a factor of \( 1 / (1 + \tau_c) \).

Defining individual’s effective savings as \( \tilde{s}_{it} = (1 + \vartheta) s_{it} \), which includes the subsidy, the government could restore individual investment to the regime of neutral innovation policy \( (\omega = 0) \) by setting the investment subsidy rate to:

\[
\vartheta = -\rho \omega \lambda / (1 - \rho \theta)
\]

where \( \vartheta > 0 \) for \( \omega < 0 \). The corresponding consumption tax is then given by:

\[
\tau_c^* = \vartheta b / (1 - b - \vartheta b)
\]

22 This tax-subsidy scheme of the government has no effect on the households’ optimal

---

21 More detailed derivations are available from the authors upon request.

22 In deriving \( \tau_c^* \), first aggregate and then substitute (27) and (12a) into (28) to get

\[
\tau_c / \vartheta = b (1 + \tau_c) / (1 - b)
\]
savings and labour supply decisions. The effective saving rate is only elevated to the point where it replicates the neutral innovation scenario. In other words, the government can mitigate (at least part of) the adverse growth effect of a progressive innovation policy by designing a tax-subsidy scheme. Our calibration exercise shows that such practice could actually lead to a possibility of an optimal progressive R&D policy. In particular, as shown in Fig. 6, growth and welfare are maximized at the lowest bound for policy, $\omega = -1.696$.

\[\gamma = \omega = -1.696\]

which solves to (31).

However, this may not be always the case, particularly, when applies to a more general utility function.
6. Conclusion

In this paper, we have developed a heterogenous-agent growth model with in-house R&D where both growth and inequality are endogenously determined. We showed that a regressive R&D policy unambiguously escalates economic inequality, and conversely. However, we found the relationship between efficiency and redistributive innovation policies to be nonlinear. Regressive innovation policies have the benefit of promoting efficiency, due to incentives on agents’ savings and effort decisions. Progressive policies also have benefits, due to imperfections in capital markets; redistributive R&D policy then provides a means to relax the credit constraints that impede certain investments. In general, the relationship between public R&D spending and welfare is hump shaped and admits an optimal degree of regressivity in public R&D spending. A case for optimal progressive R&D policy, however, has been made with a properly designed R&D policy that combines consumption tax and investment subsidy policies.

Our model is stylized and designed to contrast the effects of alternative redistributive innovation policies on inequality and efficiency, in an extreme form of market incompleteness that rules out capital markets. A simple extension of our model would be to add imperfect credit markets, by allowing individuals to have some access for credit and be able to borrow a fraction of their income, as in Aiyagari (1994). While unlikely to change the key results, a physical capital investment decision and a more general utility function can also be introduced to enrich the model further.

Acknowledgements

We have benefitted from the comments of the participants of The 2016 Africa Meeting of the Econometric Society, South Africa, 26-28 July 2016, particularly Stephen Turnovsky. We have also benefited from the seminar participants at the University of Pretoria, Durham University Business School, UNU MERIT Maastricht University and the Centre for Studies in Social Science, Calcutta. The authors are
very grateful to Gerhard Glomm for his comments on an earlier version of the paper. Changhyun Park is acknowledged for able research assistance. The usual disclaimer applies.
Appendix

A. Aggregate wealth and growth dynamics

From (13), (15) and (16), all aggregate variables except aggregate knowledge grow at the same rate:

\[ 1 + \gamma_t \equiv y_{t+1}/y_t = g_{t+1}/g_t = c_{t+1}/c_t = \Omega_t h_{t+1}/h_t \]  

(A.1)

where

\[ \Omega_t \equiv \exp \left( 0.5 (1 - \alpha) \left( \sigma_t^2 - \sigma_{t+1}^2 \right) \right) \]

Note that (A.1) holds along the transitional dynamics where inequality evolves based on its own past history as in (20). In the steady state where \( \sigma_{t+1}^2 = \sigma_t^2 = \sigma^2 \), the economy will be in a balanced growth path (BGP) where \( \gamma_t = \gamma \).

To derive the balanced growth rate, one needs to first derive the growth rate of aggregate knowledge during the transition by aggregating (17) to get the dynamics of aggregate knowledge as follows,

\[ h_{t+1} = a_3 \psi \chi h_t^{\theta + v + \lambda} \exp \left( \sigma_t^2 (\pi + F + q) \right) \]

where \( \pi, F \) and \( q \) are defined in (22). Then, given constant returns to scale in knowledge production, one obtains the steady state growth rate (21b).

B. Derivation of the steady state welfare

Note first that the discounted sum of individual welfare is simply \( V_{i0} = \max U_{i0} \). Aggregating \( V_{i0} \) across the entire population leads to the discounted sum of aggregate welfare:

\[ W_0 = \int_0^1 V_{i0} di \]

Then considering (10), we have
\[ W_0 = E_t E_0 \sum_{t=0}^{\infty} \rho^t (\ln c_t - l_t^n) \]
\[ = E_0 \sum_{t=0}^{\infty} \rho^t (\ln c_t - 0.5\sigma_t^2 - l^n) \]

Then, the steady-state discounted expected aggregate welfare is given by,

\[ W = E_0 \sum_{t=0}^{\infty} \rho^t \left( \ln \left( c_0 (1 + \gamma)^t \right) - 0.5\sigma_c^2 - l^n \right) \]
\[ = \rho / (1 - \rho)^2 \ln (1 + \gamma) + (\ln c_0 - 0.5\sigma_c^2 - l^n) / (1 - \rho) \]

where \( c_0 \) is given by, from (13) and (15),

\[ c_0 = (1 - \tau)(1 - b)la^{1/\alpha}h_0 \exp(d_0) \]

One applies similar procedures to derive the steady-state discounted expected aggregate welfare with consumption tax and investment subsidy \( (W') \):

\[ W' = \rho / (1 - \rho)^2 \ln (1 + \gamma) + (\ln c'_0 - 0.5\sigma_c^2 - l^n) / (1 - \rho) \]

where

\[ c'_0 = (1 + \tau_c)^{-1} (1 - \tau)(1 - b)la^{1/\alpha}h_0 \exp(d_0) \]

where \( \theta \) and \( \tau_c \) are given by (30) and (31), respectively.

C. Proof for Proposition 4

(i) If \( \sigma_0^2 = 0 \), which implies \( h_{t0} = h_0 \), then, from (17), \( \sigma_1^2 = \sigma^2 = \nu^2 \kappa^2 + \varphi^2 \). It is then straightforward to see \( \gamma \) increases in \( \omega \).

(ii) If \( \sigma_0^2 \neq 0 \) and \( \kappa^2 \neq 0 \) (or \( \varphi^2 \neq 0 \)), then from (17), \( \sigma^2 = \nu^2 \kappa^2 / (1 - \beta^2) \) (or \( \sigma^2 = \varphi^2 / (1 - \beta^2) \)). In this case, both \( \sigma^2 \) and \( \psi \) increase in \( \omega \) in (21b), leading to an
ambiguous effect of $\omega$ on growth.

References


