Tax Reform, Unhealthy Commodities and Endogenous Health

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Abstract

This paper explores how tax reforms with taxes on unhealthy commodities impact consumer behaviours and welfare when individual health is endogenised. We employ a dynamic general equilibrium model which includes both goods and health sectors. Although unhealthy commodities provide utility, they pose a detrimental effect on health. The analytical results show that the introduction of taxes on unhealthy commodities does not have direct effects on health in the steady state. However, based on our simulation results, with a revenue-neutral tax reform where labour income taxes are adjusted, the introduction of taxes on unhealthy commodities improves both health and welfare, but reduces leisure in the long run. On the other hand, a tax reform where capital income taxes are adjusted contributes to even higher welfare as both health and leisure improve. Our analysis may inform policy making decisions on taxation of unhealthy commodities when government can adjust pre-existing taxes.

Keywords: Unhealthy commodities taxation, endogenous health, tax reform.

JEL classification: D91, E20, H20, I18.

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1 Introduction

The rising global burden of non-communicable diseases such as diabetes, cardiovascular and coronary heart diseases, and certain types of cancer has driven policy makers to explore approaches to improve population health (Lim et al., 2013). Since many major health problems are due to individual behaviours such as over-consumption of foods and beverages high in fat, sugar and salt content, it is possible to use fiscal policy to target these unhealthy commodities (e.g. Lustig et al., 2012; Chokshi and Farley, 2014). Changing the relative prices of these commodities via taxation is one of the policies which has been proposed and explored the most in the public arena. Examples include the public health product tax in Hungary, several taxes on saturated fat in Denmark, and the recently passed Soft Drink Industry Levy (also known as the “sugar tax”) in the UK.

Taxes on unhealthy commodities should discourage consumption and thus contribute to higher level of population health. However, existing studies do not always support this intuition. Fletcher et al. (2010) find that soft drink taxes in the US induce significant changes in consumer behaviours, but the impacts on body mass index (BMI) are generally small in magnitude. In addition to the negligible impacts on health, Mytton et al. (2007) even show that the introduction of taxes on saturated fat in the UK could increase mortality due to cardiovascular diseases, because the households would increase their intake of salt. Schroeter et al. (2008) warn that people could consume other, untaxed, unhealthy commodities in response, and thus increase their BMI. Yaniv et al. (2009) explain that, even if a fat tax reduce the consumption of junk foods, obesity could still arise because the individual might spend less time exercising.

One interesting finding in the literature is that, when coupled with other fiscal instruments such as subsidies, taxes on unhealthy commodities are more likely to be beneficial to health (e.g. Nnoaham et al., 2009; Tiffin and Arnoult, 2011; Cornelsen and Carreido, 2015). Therefore, the government should consider more comprehensive policies to ensure
the positive impact on health.

One mechanism which may help explain the results is the role of health investment as in Grossman (1972) and Goulao and Pérez-Barahona (2014). Health investment, such as exercise and health care, could improve the level of health both in short and long run (e.g. Lucas et al., 2003; Nemet et al., 2005; Oja et al., 2016). This mechanism endogenises health by connecting it with certain individual behaviours. If taxes on unhealthy commodities induce the individual to invest less on health, they could fail to improve the individual’s level of health. In the canonical model of Grossman (1972), health is considered as a capital stock which increases with investment. The individual invests in health not only because it provides utility, but also because it produces more “healthy time” which is available for work. Goulao and Pérez-Barahona (2014) include the detrimental effect of unhealthy commodities into the health function of Grossman (1972), so that one can clearly identify the impacts on consumption. The result of their paper shows that taxes on unhealthy commodities, when coupled with subsidies on health investment, can restore the optimal level of health when the individuals are myopic.\footnote{O’Donoghue and Rabin (2006) and Cremer et al. (2012) also study the role of taxes on unhealthy commodities in the economy, but they do not specify the equations to illustrate the law of motion of health.}

The mechanism of health investment is also used in this paper to explore how reforms with taxes on unhealthy commodities may influence individual behaviours in relation to health. We construct a dynamic general equilibrium model with two sectors: the goods sector, which produces consumption commodities, and the health sector, which provides the individual with health. In our model, the concept of health is in line with Grossman (1972) in that health provides both utility and income benefits, so that we can clearly examine how the individual level of health is determined. This endogenous health decision is important in our paper because unhealthy commodities taxes are usually employed to target health problems which are due to consumption choices (such as lifestyle choices). Different from Goulao and Pérez-Barahona (2014), we endogenise individual income by addressing both labour supply and capital in the economy. By doing so, we are able to
highlight the roles of different fiscal instruments (taxes on labour income, capital income, and consumption), and thus the efficacy of the tax reforms. One novelty of our model is found in how it embeds individual preference for health in that of leisure. To be more specific, the individual can allocate healthy time either into leisure for higher utility or into labour supply for higher income. This novelty helps us to clarify the trade-off between leisure and labour supply when it comes to the changes in taxes on labour income. Unhealthy commodities, which also provide individual with utility, pose a detrimental effect on the accumulation of health. The individual has to find the balance between the utility and the detrimental effect from these commodities. In addition to the utility function, we also specify the production function of health with labour supply and capital. This specification allows us to carefully examine the changes in individual investment decision between the two sectors in response to the taxes. The results indicate that the implementation of taxes on unhealthy commodities does not improve the level of health directly in the steady state. Nevertheless, with revenue-neutral tax reforms which raise taxes on unhealthy commodities but lower those on income, the implementation can improve the steady state level of health and that of welfare.

The paper proceeds as follows. A two-sector model with endogenous health is introduced in Section 2. First of all, the model will be presented in a centralised economy in order to provide a more general and clearer view of the system. Individual utility and the resource constraints for the goods sector and the health sector will be carefully examined. The model will then be applied in a decentralised economy in Section 2.1, and the taxes on unhealthy commodities, numeraire commodities, capital income, and labour income will be introduced into the model. The optimisation conditions for the problem will be discussed in Section 2.2. Following those conditions, Section 2.3 will explore the steady state solutions. In Section 3, a tax analysis will be performed based on the steady state solutions solved in the previous sections. We will perform policy analysis in Section 4. Due to the complicated relationships between the variables, we calibrate the model on
the US economy in Section 4.1 to avoid ambiguous effects of the taxes. In Section 4.3, we will employ the calibrated parameters to analyse the impacts of two revenue-neutral tax reforms: the reform which levies unhealthy commodities taxes whilst adjusting labour income taxes and the reform which levies unhealthy commodities taxes whilst adjusting capital income taxes. Finally, conclusions will be offered in Section 5.

2 The Model

This section presents the model for the analysis. In the model, the individual can freely allocate healthy time into leisure or labour supply. Healthy time can be attained through the accumulation of health, \( h \), with decreasing marginal returns (Grossman, 1972). The individual determines \( l \) fraction of healthy time spent on labour supply and leaves \((1 - l)\) for leisure. Along with leisure, the numeraire goods, \( c \), and unhealthy commodities, \( x \), also bring the individual with utility. The individual lifetime utility is as follows:

\[
U = \int_{0}^{\infty} u(c, x, L) e^{-\rho t} dt,
\]

(1)

where \( L \equiv (1 - l)h^{\mu} \) is leisure, \( h^{\mu} \) is healthy time, and \( \rho \) is the rate of time preference.

The economy is constituted by the goods sector \( y \) and the health sector \( m \), and both sectors require the same inputs of capital and labour supply following the setups of Bond et al. (1996) and Azariadis et al. (2013).

\[
y = f(sk, vlh^{\mu}), \]

(2)

\[
m = m((1 - s)k, (1 - v)lh^{\mu}), \]

(3)

where \( s \) and \( v \) are the fractions of capital and labour supply devoted into the goods sector.

The prices of the two goods are standardised into unity for simplicity. The law of
motion in the capital thus is set as follows:

\[ \dot{k} = y - c - x, \]  

(4)

where the variables with a dot on the top hereafter represents the growth of that variable.

\( x \) enters the law of motion of \( h \) because it poses detrimental effects on health. In consistent with Goulao and Pérez-Barahona (2014), the natural depreciation of health, \( \delta \), is not affected by the consumption of \( x \):

\[ \dot{h} = m - \eta x - \delta h, \]  

(5)

where \( \eta \geq 0 \) and \( \delta \geq 0 \). In this expression, \( \eta \) is the measure of the detrimental effects of unhealthy commodities on health. The extreme case of \( \eta = 0 \) refers to the situation where unhealthy commodities pose negligible effects on health.

2.1 The Decentralised Economy

In a decentralised economy, the firms in both sectors seek to maximise their own profits. The firm in the goods sector sells its production \( y \) and pays the rental prices of \( r_y \) and \( w_y \) for the capital and labour supply that it uses.

\[
\max. \; y - r_y sk - w_y vlh^n, 
\]  

(6)

where

\[ r_y = f_1, \]  

(7a)

\[ w_y = f_2. \]  

(7b)
In this expression, \( f_1 \) and \( f_2 \) are the marginal product of capital and that of labour supply. In the health sector, the producer sells \( m \) with the price of \( p_m \), and pays the rental prices of \( r_m \) and \( w_m \) for the capital and labour supply used in production. The optimisation problem can be written as:

\[
\max p_m m - r_m (1 - s) k - w_m (1 - v) lh^\mu,
\]

where

\[
r_m = p_m m_1, \quad (9a)
\]

\[
w_m = p_m m_2. \quad (9b)
\]

\( m_1 \) and \( m_2 \) are the marginal product of capital and that of labour supply in the health sector.

At every point of time, the government receives tax revenue from taxes on capital income, labour income, and commodities. Assuming the government balances its budget by financing a lump-sum transfer, \( G \), to the individual:

\[
G = \tau_k (sr_y + (1 - s)r_m)k + \tau_l (vw_y + (1 - v)w_m)lh^\mu + \tau_c c + \tau_x x,
\]

where \( \tau_k, \tau_l, \tau_c, \) and \( \tau_x \) are the taxes on capital income, labour income, numeraire goods, and unhealthy commodities.

One can view \( p_m m \) and the taxes paid in the health sector as forgone opportunities to accumulate more capital. Therefore, equation (4) can then be transformed into:

\[
\dot{k} = (1 - \tau_k)(r_y sk + r_m (1 - s) k + (1 - \tau_l)(w_y v lh^\mu + w_m (1 - v) lh^\mu) - (1 + \tau_c)c - (1 + \tau_x)x - p_m m + G. \quad (11)
\]
2.2 The Optimisation Problem

Maximising the utility (1) under the constraints of (5) and (11), we form the Hamiltonian function as follows:

$$\mathcal{H} = u(c, x, L) + \lambda[(1 - \tau_k)(r_y s k + r_m(1 - s)k) + (1 - \tau_l)(w_y v l h^\mu + w_m(1 - v)l h^\mu)]$$

$$- (1 + \tau_c)c - (1 + \tau_x)x - p_m m + G] + q[m - \eta x - \delta h],$$

where $\lambda$ is the shadow price of capital, and $q$ is the shadow price of health. The first-order conditions for this optimisation problem are:

$$u_c = \lambda(1 + \tau_c), \quad (12a)$$
$$u_x = \lambda(1 + \tau_x) + q\eta, \quad (12b)$$
$$u_L h^\mu = \lambda(1 - \tau_l)(w_y v l h^\mu + w_m(1 - v)h^\mu), \quad (12c)$$
$$r_y = r_m, \quad (12d)$$
$$w_y = w_m, \quad (12e)$$
$$\lambda p_m = q, \quad (12f)$$
$$\lambda(1 - \tau_k)(r_y s + r_m(1 - s)) = \lambda \rho - \dot{\lambda}, \quad (12g)$$
$$\mu u_L (1 - l) h^{\mu - 1} + \lambda \mu(1 - \tau_l)(w_y v l h^{\mu - 1} + w_m(1 - v)l h^{\mu - 1}) - q\delta = q \rho - \dot{q}, \quad (12h)$$

along with the transversality conditions,

$$\lim_{t \to \infty} e^{-\rho t} \lambda(t)k(t) = 0, \quad (12i)$$
$$\lim_{t \to \infty} e^{-\rho t} q(t)h(t) = 0. \quad (12j)$$

(12a) equates the optimal consumption of numeraire goods to the product of shadow price of $k$ and $(1 + \tau_c)$; (12b) shows that the optimal consumption of unhealthy commodities
partly depends on both the shadow price of \( k \) and that of \( h \); (12c) equates the marginal utility of leisure to the marginal costs of labour supply in both sectors; (12d) and (12e) describe the optimal allocation of inputs between the two sectors; (12f) implies that the relative value of the two shadow prices depends on \( p_m \); (12g) and (12h) are the Euler equations; and (12i) and (12j) are the conditions to exclude any Ponzi game in the economy.

With (12d), we can reform the evolution of \( \lambda \) from (12g) as:

\[
\frac{\dot{\lambda}}{\lambda} = \rho - (1 - \tau_k) r,
\]

where \( r \equiv r_y = r_m \). With (12c), (12e), and (12f), the evolution of \( q \) in (12h) can be written as:

\[
\frac{\dot{q}}{q} = \rho + \delta - \frac{\mu}{p_m} (1 - \tau_l) w h^{\mu - 1},
\]

where \( w \equiv w_y = w_m \).

### 2.3 The Equilibrium

In this subsection, the steady state solutions will be explored by using the first-order conditions presented in Section 2.2. As shown in (13), \( r \) is pinned down by \( \rho \) and \( \tau_k \) in the steady state. If the production technology in the goods sector is constant returns to scale, \( \frac{\nu l h^p}{k} \) and \( \frac{r_y}{w_m} \) are fixed in the steady state. Accordingly, referring to equations (12d) and (12e), \( \frac{m_1}{m_2} \) and thus \( p_m \) are also pinned down in the steady state.

With (7a), (7b), (12g), and (12h), we can then attain the steady state level of \( h \) as follows:

\[
h^* = \left( \frac{f_2 \mu (1 - \tau_l) \rho}{f_1 p_m (\rho + \delta)(1 - \tau_k)} \right)^{\frac{1}{\tau_x}}.
\]

Given a constant labour to capital ratio in either sector, the level of \( h \) converges to an optimum in the long run as shown in (15). The somewhat counter-intuitive fact that \( \tau_x \) does not appear in the equation implies the following proposition:
Proposition 1. With a constant labour to capital ratio, taxes on unhealthy commodities do not affect the level of health in equilibrium.

The main reason is that $m$ has to decrease in response to the decrease in $x$, so that the steady state condition for (5) can be held. The story behind this logic is that the individual finds it beneficial to decrease the investment in health, $m$, when the detrimental effects on $h$ diminishes. Therefore, reduced health investment offsets the beneficial effect of reducing of $x$, leaving $h^*$ unchanged. However, (15) only implies that the implementation of $\tau_x$ would have no direct impacts on the level of health. It is possible that $\tau_x$ can still affect the level of health through indirect channels with more complete tax reforms. Detailed information of the tax reform will be provided in Section 4.3.

Equations (15) is derived from general functions. To provide a clearer view of the following analysis, we adopt specific functions for (1)-(3). The utility function (1) is set to be the following form:

$$u(c, x, L) = \ln c + \theta \ln x + \psi \ln((1 - l)h^\mu),$$

where $\theta$ is the preference to unhealthy commodities, $x$, and $\psi$ is the preference to leisure. The production functions follow the Cobb-Douglas forms:

$$y = A(sk)^\alpha(vlh^\mu)^{(1 - \alpha)},$$

$$m = B((1 - s)k)^\beta((1 - v)lh^\mu)^{(1 - \beta)},$$

where $\alpha$ and $\beta$ are the shares of capital and $A$ and $B$ are the production efficiency factors in the two sectors.

With the specified production functions, we are able to clarify the relationship between
s and v by using (12d) and (12e):

\[ v = \frac{\beta(1 - \alpha)s}{\alpha(1 - \beta) - (\alpha - \beta)s}, \quad (19) \]

which implies that v is increasing in s when \( \alpha \geq \beta \). This condition is also shown in Azariadis et al. (2013). If the capital and labour supply are complements, then the allocation of capital in one sector should be positively related to that of labour supply. Note that, when \( \alpha = \beta, v = s \), meaning that the individual invests in the goods sector with identical fractions of both inputs.

Considering the evolution of \( \lambda \) will stop in the steady state, we derive the following condition from (7a), (13), and (17):

\[ \frac{lh^\mu}{k} = \frac{s}{v} \left( \frac{\rho}{\alpha A(1 - \tau_k)} \right)^{\frac{1}{\alpha - \beta}}. \quad (20) \]

The left-hand side of the equation is the labour to capital ratio in the economy. Equation (20) shows that this ratio is partly determined by the ratio of \( s \) to \( v \). In addition to \( \dot{\lambda} = 0, \dot{q} \) should also be zero in the steady state. Therefore, with (7b), (14), and (18), we derive that:

\[ \frac{lh^\mu}{k} = \frac{s}{v} \left( \frac{\mu A(1 - \tau_l)(1 - \alpha)}{p_m(\delta + \rho)} \right)^{\frac{1}{\alpha}} h^{\frac{1-\mu}{\alpha}}. \quad (21) \]

With (20) and (21), we can characterise \( h^* \) as:

\[ h^* = \left( \frac{\alpha A(1 - \tau_k)}{\rho} \right)^{\frac{\mu}{1-\alpha}} \left( \frac{\mu A(1 - \tau_l)(1 - \alpha)}{p_m(\delta + \rho)} \right)^{\frac{1}{\alpha - \beta}}. \quad (22) \]

We can also attain a specified function for \( p_m \) with (9a), (13), and (20):

\[ p_m = \frac{\alpha A^{\frac{\mu}{1-\alpha}}}{\beta^\beta(1 - \beta)} \left( \frac{1 - \alpha}{1 - \beta} \right)^{1-\beta} \left( \frac{A^{\frac{1-\beta}{1-\alpha}}}{B} \right)^{\frac{1-\beta}{1-\alpha}} \left( \frac{1 - \tau_k}{\rho} \right)^{\frac{\alpha - \beta}{1-\alpha}}. \quad (23) \]

This equation implies that the relative price of \( m \) is affected by the ratio of the production
efficiency factors in both sectors. Given the inputs in both sector, a higher $A$ contributes a more efficient production in the goods sector compared to that in the health sector, and thus decrease the relative prices of the products in the goods sector. Therefore, $p_m$ increases as $A$ increases. On the other hand, if the production in the health sector is more efficient than the production in the goods sector, relative prices of the products in the goods sector are expected to be higher. Therefore, $p_m$ decreases as $B$ raises the relative productivity in the health sector. Equation (23) also implies a negative relationship between $\tau_k$ and $p_m$. The reason behind this negative relationship is that larger taxes on capital income reduce the after-tax marginal product of capital (as in (12g)) and thus decrease the relative shadow price of $q$ to $\lambda$. Based on the positive relationship between $p_m$ and the ratio of $q$ to $\lambda$ shown in (12f), an increase in $\tau_k$ thus decreases $p_m$ in the long run.

The steady state of $k$ can be obtained by using (20) and (22):

$$k^* = \frac{v^*}{s^*} \left( \frac{\alpha A (1 - \tau_k)}{\rho} \right)^{\frac{1}{1-\alpha}} l^*(h^*)^\mu,$$

(24)

where $k^*$ has to increase as the labour supply increases, $l^*(h^*)^\mu$, so that the labour to capital ratio is fixed in equilibrium.

We then derive $x$ as a function of $c$ by rewriting (12b) with the specification in (16).

$$x^* = \frac{\theta (1 + \tau_c)}{1 + \tau_x + p_m \eta} c^*.$$

(25)

This equation shows that the consumption choice of $x$ over $c$ is positively affected by $\tau_c$ and the measure of the individual preference to unhealthy commodities, $\theta$. Given $c^*$, equation (25) also indicates that $\frac{\partial x^*}{\partial \tau_x} < 0$, $\frac{\partial x^*}{\partial \eta} < 0$, and $\frac{\partial x^*}{\partial p_m} < 0$. In accordance to the prevailing hypothesis of the supporters of taxes on unhealthy commodities, $\tau_x$ deters the consumption of unhealthy commodities because it raises the relative prices of unhealthy commodities. To understand the negative effects of $\eta$: $\eta$ is the marginal detrimental effect of consuming one unit of $x$ (as in (5)). Therefore, given all other parameters, an increase in $\eta$ would
make the individual less willing to consume $x^*$ due to the increased marginal costs. In terms of the negative relationship between $x^*$ and $p_m$, an increase in $p_m$ indicates that the individual has to pay more to compensate for the loss in health. Therefore, given all other parameters, the individual would reduce $x^*$ in response to the increase in $p_m$.

Next, we rewrite (12c) into:

$$c^* = \frac{p_m(\rho + \delta)}{\mu \psi(1 + \tau_c)} (1 - l^*) h^*. \tag{26}$$

In addition to the evolutions of $\lambda$ and $q$, the evolution $h$ should also be zero in the steady state. Therefore, we force $\dot{h} = 0$ in (5) and obtain that:

$$m = \eta x + \delta h. \tag{27}$$

With (18) and (21), and (25), equation (27) can be written as:

$$\frac{\rho + \delta}{\mu (1 - \beta)(1 - \tau_l)} (1 - v) l h = \frac{\eta \theta (1 + \tau_c)}{1 + \tau_x + p_m \eta} c + \delta h. \tag{28}$$

With (20) and (25), the market clearing condition in the goods sector, $y = x + c$, can be reformed into:

$$\frac{p_m(\rho + \delta) v l h}{\mu (1 - \tau_l)(1 - \alpha)} = \frac{(\pi + \theta (1 + \tau_c)) c}{\pi}. \tag{29}$$

Equations (22), (26), (28), and (29) form a system which could be used to solve for the steady state of $c$, $l$, and $v$:

$$c^* = \frac{\omega (1 - \tau_l)(1 - l^*) h^*}{\psi (1 + \tau_c)}, \tag{30}$$

$$l^* = \frac{p_m(1 - \beta)(\delta \pi h^* + \eta \theta (1 + \tau_c) c^*)}{\omega (1 - v^*) \pi h^*}, \tag{31}$$

$$v^* = \frac{(1 - \alpha)(\pi + \theta (1 + \tau_c)) c^*}{\pi \omega l^* h^*}. \tag{32}$$
where $\omega \equiv \frac{p_m(p + \delta)}{\rho(1 - \tau)}$ and $\pi \equiv 1 + \tau_x + p_m'$. With (19), one can also attain a function for $s^*$:

$$s^* = \frac{\alpha(1 - \beta)(\pi + \theta(1 + \tau_c))c^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta\pi\omega l^*h^*} \quad (33)$$

3 Tax Analysis

This section conducts a comparative static analysis to examine the long-run impacts of taxes on unhealthy commodities. To investigate the impacts of $\tau_x$ on the variables, we perform comparative static analysis based on the steady state solutions. Referring to (30), $\tau_x$ affects $c^*$ through the following channel:

$$\frac{dc^*}{d\tau_x} = \frac{\partial c^*}{\partial l^*} \frac{dl^*}{d\tau_x} > 0. \quad (34)$$

Since $\frac{dc^*}{d\tau_x} = \frac{c^*}{\tau_x} < 0$, the positive derivative of (30) with respect to $\tau_x$ implies that $\frac{dl^*}{d\tau_x} < 0$. The above derivation shows that changes in $\tau_x$ do not affect $c^*$ directly; instead, they affect $c^*$ via their impacts on $l^*$. The decrease in $l^*$ implied by (34) would result in two opposing effects on $c^*$: first, it increases leisure, so the individual has to increase $c^*$ to maintain the marginal rate of substitution (MRS) between $c^*$ and leisure; second, it decreases labour supply, so the individual has to decrease $c^*$ to hold $\dot{k} = 0$ in the steady state. In our model, the first effect dominates the second effect, so the overall effect of $\tau_x$ on $c^*$ is positive.

As stated in (19), $s$ and $v$ move in the same direction as long as $\alpha \geq \beta$. If the parameters comply with this condition, the changes in $s^*$ would automatically imply the changes in $v^*$. For succinctness, the following analysis will just focus on the transitions in $s^*$. As shown in (33), the changes in $\tau_x$ affect $s^*$ both directly and indirectly:

$$\frac{ds^*}{d\tau_x} = \frac{\partial s^*}{\partial c^*} \frac{dc^*}{d\tau_x} + \frac{\partial s^*}{\partial l^*} \frac{dl^*}{d\tau_x} + \frac{\partial s^*}{\partial \tau_x} \geq 0. \quad (35)$$

In the above expression, $\frac{\partial s^*}{\partial c^*} = \frac{\beta\pi\omega l^*h^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta\pi\omega l^*h^*} > 0$, $\frac{\partial s^*}{\partial l^*} = \frac{-\beta\pi\omega h^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta\pi\omega l^*h^*} < 0$, $\frac{\partial s^*}{\partial \tau_x} = \frac{-\beta\pi\omega \omega^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta\pi\omega l^*h^*} < 0$, $\frac{\partial s^*}{\partial \tau_x} =$
An increase in \( c^* \) induced by \( \tau_x \) would force \( s^* \) to increase so that the goods market clearing condition can be satisfied. In response to the negative impact of \( \tau_x \) on \( l^* \) (which is implied in (34)), the individual has to increase \( s^* \) to hold the MRS between consumption and leisure constant. The illustration for the negative direct effect of \( \tau_x \) on \( s \) is that, as shown in the resource constraint of (11), larger \( \tau_x \) reduce the accumulation of \( k \) because they crowd out the resources available. It should be noted that the direct and indirect effects of \( \tau_x \) on \( s^* \) are opposite to each other, and we are not able to determine which effect dominate the others. Specific parameters are required to solve this complex interrelationship. With this issue in mind, the calibration and a more specific analysis will be presented in Section 4.1.

Referring to (31), the negative relationship between \( \tau_x \) and \( l^* \) can be disentangled into the following form:

\[
\frac{dl^*}{d\tau_x} = \frac{\partial l^*}{\partial c^*} \frac{dc^*}{d\tau_x} + \frac{\partial l^*}{\partial v^*} \frac{dv^*}{ds^*} \frac{ds^*}{d\tau_x} + \frac{\partial l^*}{\partial \tau_x} < 0,
\]

(36)

where \( \frac{\partial l^*}{\partial c^*} = \frac{\eta \theta (1+\tau_x)c^*}{\eta \theta (1+\tau_x)c^* + \alpha} > 0 \), \( \frac{\partial l^*}{\partial v^*} = \frac{r}{1-v} > 0 \) and \( \frac{\partial l^*}{\partial \tau_x} = \frac{-\eta \theta (1+\tau_x)c^*}{\pi (\beta \omega + \eta \theta (1+\tau_x)c^*)} < 0 \). As shown in (34), larger \( \tau_x \) increase \( c^* \). This positive impact on \( c^* \) translates into two opposing effects on \( l^* \): first, \( l^* \) has to increase to hold the goods market clearing condition; second, \( l^* \) has to decrease to restore the MRS between consumption and leisure. However, the first effect dominates the second one, so the indirect effect of \( \tau_x \) through \( c^* \) is positive on \( l^* \). Due to the ambiguous effect of \( \tau_x \) on \( s^* \), we cannot determine whether the indirect effect is positive or negative.

On the other hand, the direct effect of \( \tau_x \) on \( x^* \) is negative in that an increase in \( \tau_x \) would violate the steady state condition for (11). Therefore, the individual has to decrease the input in the production function \( y \). Although the indirect effect is ambiguous in the general case, the overall effect of implementing \( \tau_x \) is negative to \( l^* \).

The changes in \( \tau_x \) would change \( x^* \):

\[
\frac{dx^*}{d\tau_x} = \frac{\partial x^*}{\partial c^*} \frac{dc^*}{d\tau_x} + \frac{\partial x^*}{\partial \tau_x} < 0,
\]

(37)
where $\frac{dx^*}{dc^*} = \frac{c}{x} > 0$ and $\frac{dx^*}{\tau_x} = \frac{x}{\tau_x} < 0$. The expression above shows that, in addition to the direct channel, $\tau_x$ can also impact $x^*$ through the indirect channel of $c^*$. The illustration for $\frac{dx^*}{dc^*} > 0$ is that the individual has to consume more $x$ with increased $c$ due to larger $\tau_x$, so that the MRS between $x$ and $c$ remains constant (as in (12b)). Referring to the positive relationship between $c^*$ and $\tau_x$ discussed in (34), the indirect effect of $\tau_x$ is positive on $x^*$. The illustration of the direct effect is straightforward: higher prices on $x$ deter the consumption of $x$. Based on the full derivative, the direct effect of $\tau_x$ on $x^*$ outweighs the indirect effect. This negative relationship between $x^*$ and $\tau_x$ is in line with the proposition of most of the supporters for taxes on unhealthy commodities.

Intuitively, since $\tau_x$ reduces $x^*$, larger $\tau_x$ should be beneficial to $h^*$. However, as shown in (22), it is surprising that $\tau_x$ plays no role in $h^*$. As explained in Section 2.3, although $\tau_x$ reduces the consumption of $x$, the individual has to reduce the investment in the health sector in order to hold the optimality condition in (5). The decreased investment in the health sector offsets the positive force from the reduced $x^*$. On the other hand, $h^*$ can be affected by the implementations of $\tau_l$ and $\tau_k$:

$$\frac{dh^*}{d\tau_l} = \frac{\partial h^*}{\partial \tau_l} < 0,$$  \hspace{1cm} (38)
$$\frac{dh^*}{d\tau_k} = \frac{\partial h^*}{\partial \tau_k} + \frac{\partial h^*}{\partial p_m} \frac{\partial p_m}{\partial \tau_k} < 0,$$  \hspace{1cm} (39)

where $\frac{\partial h^*}{\partial \tau_l} = \frac{-h^*}{(1-p)(1-\tau_l)} < 0$, $\frac{\partial h^*}{\partial \tau_k} = \frac{-\alpha h^*}{(1-\alpha)(1-p)(1-\tau_k)} < 0$, and $\frac{\partial h^*}{\partial p_m} = \frac{-h^*}{(1-p)p_m} < 0$. It is worth noting that the direct effects of income taxes on health are both negative. The reason is that the implementation of either taxes would immediately crowd out the resources available for the health sector. In addition to the direct effect, changes in $\tau_k$ also affect $h^*$ through the channel of $p_m$. $h^*$ is negative in $p_m$ because higher prices on health services deter the investment in the health sector. Considering the negative relationship between $p_m$ and $\tau_k$, this indirect effect is thus positive. Nevertheless, the direct effect of $\tau_k$ outweighs the indirect effect, so the relationship between $h^*$ and $\tau_k$ is still negative.
4 Policy Analysis

Due to the complexity of the comparative-static effects, the analysis of the tax reform requires specific parameters for unambiguous variations in the variables. We first apply the model to the US economy, and then simulate the changes of the economy with two proposed reforms under the revenue-neutral scheme: the reform which adjusts $\tau_l$ to offset the change in revenue induced by $\tau_x$, and the reform which adjusts $\tau_k$ to offset the changes in revenue induced by $\tau_x$.

4.1 Calibration

Although we calibrate the model on the US economy, we have to employ the Canadian data for the natural depreciation of health, $\delta$. Considering the similarity in their natural force of health depreciation, the data of Canadian population can be a good approximation for the US population (e.g. Rockwood and Mitnitski, 2007; Dalgaard and Strulik, 2014; Strulik, 2015). The value $\delta$ is selected as 0.043 based on the estimation of Mitnitski et al. (2002).

Referring McDaniel (2007), the average tax rates on labour income, capital income, and commodities in the US from 1970-2013 are around 0.20, 0.28, 0.08 respectively. Therefore, the initial tax rates are set to be $\tau_l = 0.20$, $\tau_k = 0.28$ and $\tau_c = \tau_x = 0.08$. The rate of time preference, $\rho$, is selected to be 0.04 following Azariadis et al. (2013). The share of capital in the goods sector, $\alpha$, is set to be 0.3 as in Chen and Lu (2013). We employ the data of $\beta = 0.22$ in the health industry documented by Acemoglu and Guerrieri (2008). It is worth noting that, as implied by (19), the observation of $\alpha = 0.3$ and $\beta = 0.22$ entails $v'(s) \geq 0$.

Prescott (2006) pointed out that the fraction of time allocated to labour market in the US is around 25 percent, so we choose $l^*$ to be 0.25 in this paper. The initial output in the goods sector, $y$, is normalised to 1, so that all the other economic variables can be easily presented as a fraction of $y$. Data retrieved from the Consumer Survey conducted by
Bureau of Labor Statistics shows that the expenditure on food-away-from-home 2013-2015 is around 4% as a ratio of total household expenditure.\footnote{The survey items partly changed in 2013, so we only include the data 2013-2015 to maintain the consistency.} As shown in (11), the GDP in this model is \( y + p_mm \), so the consumption of unhealthy commodities as a ratio of GDP is then calibrated in the following form:

\[
\frac{x^*}{y + p_mm} = 0.04.
\]

The OECD statistics shows that the ratio of health expenditure to GDP in the US between 1970 and 2013 is around 12%. This information can be employed into the calibration in the form of:

\[
\frac{p_mm}{y + p_mm} = 0.12.
\]

With the normalisation of \( y \), the ratio of \( p_mm \) to \( y \) is 0.1364. This value then implies that the fraction \( x^* \) to \( y \) should be 0.0455. With the goods market clearing condition, the fraction of \( c^* \) to \( y \) is then be calibrated as 0.9545. Equations (12d) and (12e) imply that, with the specified parameters, \( s \) and \( v \) can be calibrated as 0.9091 and 0.8681 respectively.

By using (7b), (9b), and (12e), we rewrite (12c) in the form of:

\[
\psi = \frac{(1 - \tau_l)(1 - l^*)(\frac{y}{c^*})(1 - \alpha) + (1 - \beta)p_mm}{y}. \tag{40}
\]

Inserting the specified parameters into the above equation, we then calibrate that \( \psi = 1.8772 \). With (12g), (7a), (9a), and (12d), the ratio of \( k \) to \( y \) can then be calibrated as 5.9400.

The determination of \( p_m \) is relatively flexible, because a different \( p_m \) could be the result of health services being calculated in different units. In this paper, we select \( p_m \) to be 1 for simplicity. This selection further entails \( \lambda = q \) as in (12f). With (12c) and (12h), the value of \( h^* \) can be presented as:

\[
h^* = \frac{\mu \psi (1 + \tau_c)c^*}{(1 - l^*)(\rho + \delta)}, \tag{41}
\]
which indicates that the value of $h$ in this calibration is affected by the value of $\mu$.

In the steady state, $\dot{h} = 0$. By using (41), we rewrite (5) into:

$$\eta = \frac{y}{x^*} \left( \frac{m}{y} - \frac{\mu \psi(1 + \tau_c)c^*}{(1 - l^*)(\rho + \delta)y} \right), \quad (42)$$

indicating that the determination of $\mu$ affects the value of $\eta$. For $\eta$ to be non-negative, the term inside the bracket of equation (42) has to be greater or equal to zero. Therefore,

$$\mu \leq \frac{(1 - l^*)(\rho + \delta)m}{\delta \psi(1 + \tau_c)c^*}, \quad (43)$$

where the expression on the right-hand side of the equation is the upper limit of $\mu$, $\bar{\mu}$. With the specific parameters presented in this paper, $\bar{\mu} = 0.1020$. This paper chooses $\mu = 0.09$ as the benchmark. With the benchmark $\mu$, $\eta$ is then 0.0632, and the ratio of $h$ to $y$ is 2.7980. $\theta$ can thus be calibrated as 0.0347 from (12b). By using (2) and (3), we then calculate that $A = 1.6466$ and $B = 2.0797$. In addition to the benchmark parameter set, we also include the other two parameter sets:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter set 1 (benchmark)</th>
<th>Parameter set 2</th>
<th>Parameter set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.09</td>
<td>0.102</td>
<td>0.04</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0632</td>
<td>0.0476</td>
<td>0.128</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3531</td>
<td>0</td>
<td>1.8236</td>
</tr>
<tr>
<td>$A$</td>
<td>1.6466</td>
<td>1.6178</td>
<td>1.7461</td>
</tr>
<tr>
<td>$B$</td>
<td>2.0797</td>
<td>2.0394</td>
<td>2.2203</td>
</tr>
</tbody>
</table>

It should be noted that the parameter set 2 is the extreme case where unhealthy commodities do not post any detrimental effect on health ($\mu = \bar{\mu}$). In addition, we choose parameter set 3 to consider the case where the detrimental effect of unhealthy commodities is higher.
4.2 Comparative-Static Analysis of Changing $\tau_x$ Alone

We present the simulation results in Figure 1 to provide a clearer view of the effects of implementing $\tau_x$ alone.

![Figure 1: Comparative-static effects of $\tau_x$](image)

This figure shows the comparative-static effects of $\tau_x$ in the steady state with three parameter sets listed in Table 1: (1) The solid curves denote the case with benchmark parameter; (2) the dashed curves denote the case with parameter set 2; (3) the dash-dotted curves denote the case with parameter set 3. It is clear that the effects of $\tau_x$ on $s^*$ are somewhat different across scenarios: $s^*$ slightly decreases as $\tau_x$ increases when $\mu = \bar{\mu}$, whilst it increases when $\mu < \bar{\mu}$. In the case of $\mu = \bar{\mu}$, unhealthy commodities act similarly to numeriare goods in that they both provide the individual with utilities but not posing negative impacts on health. Consequently, the individual would not have the incentive to adjust the amount of investment in health in response to the changes in $x^*$. Considering the
decreases in both labour supply and capital, the individual has to move the investments away from the goods sector to restore the production in the health sector. The effects of $\tau_x$ on $m$ are plotted in Figure 2:

![Figure 2: The effects of $\tau_x$ on $m$](image)

Figure 2 clearly shows that in the case of $\mu = \bar{\mu}$, health investment is basically fixed regardless of the changes in $\tau_x$. However, when $\mu \leq \bar{\mu}$, the individual decreases the overall $m$ in response to the increases in $\tau_x$.

To simplify the discussion, this paper will focus more on the benchmark scenario. The results of the comparative static analysis are listed in the Table 2.
Table 2: Changes in tax rates with government revenue being inconsistent

<table>
<thead>
<tr>
<th>τ_j</th>
<th>τ_k</th>
<th>τ_x</th>
</tr>
</thead>
<tbody>
<tr>
<td>c^*</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>x^*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>l^*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>s^*</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>k^*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>h^*</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

The simulation for the comparative-static effects of the income taxes can be found in Appendix B. It should be noted that the variations listed in Table 2 are in the context of government revenue being inconsistent. In the next subsection, we will introduce tax reforms with revenue neutrality.

4.3 Tax Reform

The derivation of this subsection starts from the simplification of the government budget (10).

\[ F \equiv \tau_k rk + \tau_l wh + \tau_x x - G = 0. \]  \hspace{1cm} (44)

With the calibration discussed in Section 4.1, it is clear that \( \frac{dF}{d\tau_k} > 0, \frac{dF}{d\tau_l} > 0, \frac{dF}{d\tau_x} > 0, \frac{dF}{d\tau_c} > 0. \) Employing the implicit function theorem, the balanced government budget leads to:

\[ \frac{d\tau_k}{d\tau_x} = -\frac{dF/d\tau_x}{dF/d\tau_k} < 0, \text{ and } \frac{d\tau_l}{d\tau_x} = -\frac{dF/d\tau_x}{dF/d\tau_l} < 0. \]

These negative relationships confirm that the government can keep revenue constant by raising \( \tau_x \) whilst deducing \( \tau_l \) or \( \tau_k \). Therefore, we propose two potential tax reforms: first, raising \( \tau_x \) whilst reducing \( \tau_l \); second, raising \( \tau_x \) whilst reducing \( \tau_k \). To calculate appropriate income tax rates, we endogenise \( \tau_l \) and \( \tau_k \) in this subsection an plot the income tax rates in response to the changes in \( \tau_x \) in Figure 3.
Figure 3: The replacement of $\tau_l$ and that of $\tau_k$ with $\tau_x$ under revenue-neutral schemes

In Figure 3, the solid curve indicates the changes in the average rate of labour income tax in response to the increases in $\tau_x$ with revenue neutrality, and the dashed curve represents those in the average rate of capital income tax in response to the increases in $\tau_x$ with revenue neutrality. In the following analysis, the model is applied to the simulation by changing $\tau_x$ from 0% to 100% to examine the long run effects of the tax reforms.\(^3\)

\(^3\)For the concision of the paper, the analysis here only includes the benchmark scenario. The changes in other scenarios are presented in Appendix C
The variations of the variables at the steady state due to the replacement of $\tau_k$ with $\tau_x$ is presented in Figure 5.

The effects of the tax reforms of replacing $\tau_l$ or $\tau_k$ with $\tau_x$ on $c^*$ can be separated into
two parts:
\[
\frac{dc^*}{d\tau_x} = \frac{\partial c^*}{\partial \tau_x} + \frac{\partial c^*}{\partial \tau_i} \frac{d\tau_i}{d\tau_x}, \text{ where } i = l, k.
\] (45)

As shown in Table 2, it is known that \(\frac{\partial c^*}{\partial \tau_x} > 0\), \(\frac{\partial c^*}{\partial \tau_l} < 0\), and \(\frac{\partial c^*}{\partial \tau_k} < 0\). Based on the analysis in Section 3, larger \(\tau_x\) contribute to a higher \(c^*\) through the channel of \(l^*\). In addition to this effect, increases in \(\tau_x\) would also affect \(c^*\) through the channel of decreasing \(\tau_l\) or that of \(\tau_k\) with the reforms. To illustrate the channel of decreasing \(\tau_l\): a decrease in \(\tau_l\) raises the marginal cost of leisure (as shown in (12c)). This effect would encourage the individual to provide more labour supply, resulting in two opposing effects. First, it contributes to a higher output in the goods sector. To clear the goods market, the consumption of \(c^*\) (and also \(x^*\)) has to increase in the long run. Second, it decreases leisure, so that the individual has to increase \(c^*\) to hold the MRS constant. As shown in Figure 4, the negative effect is overshadowed by the positive effects, resulting in a case where \(c^*\) is always increasing in \(\tau_x\). The channel of decreasing \(\tau_k\) can be examined from (12g): a decrease in \(\tau_k\) raises the after-tax marginal product of capital, so an increase in \(k^*\) is needed to reduce the pre-tax marginal product of capital. As a result, \(c^*\) has to increase so that the steady state condition for the resource constraint can be held.

Figures 4 and 5 show that \(x^*\) decreases in \(\tau_x\) in both reforms. The reason behind these relations can be seen from equation (25):
\[
\frac{dx^*}{d\tau_x} = \frac{\partial x^*}{\partial \tau_x} + \frac{\partial x^*}{\partial \tau_i} \frac{d\tau_i}{d\tau_x},
\] (46)

where \(\frac{\partial x^*}{\partial \tau_x} < 0\), \(\frac{\partial x^*}{\partial \tau_l} < 0\), and \(\frac{\partial x^*}{\partial \tau_k} < 0\). The full derivative shows that the negative relationship between \(\tau_x\) and \(x^*\) is composed of two opposite effects: the negative effect of \(\tau_x\) and the positive effect through the channel of decreasing \(\tau_l\) or \(\tau_k\). The negative effect of \(\tau_x\) can be examined from (37). To understand the positive effects through the channels of decreasing income taxes, we note that any changes in \(\tau_l\) or \(\tau_k\) which affects \(c^*\) would require the adjustment in \(x^*\) so as to restore the MRS. Therefore, an increase in \(c^*\) induced by smaller
\( \tau_l \) would prompt the individual to raise \( x^* \); likewise, an increase in \( c^* \) due to smaller \( \tau_k \) would also make the individual increase \( x^* \). However, with the calibration in Section 4.1, the positive effect always dominates the negative effects through income taxes in both reforms.

To explain the difference between the impacts of the reforms on \( l^* \), we first take the full derivatives of \( l^* \) with respect to \( \tau_x \):

\[
\frac{dl^*}{d\tau_x} = \frac{\partial l^*}{\partial \tau_x} + \frac{\partial l^*}{\partial \tau_l} \frac{d\tau_l}{d\tau_x},
\]

(47)

where \( \frac{\partial l^*}{\partial \tau_x} < 0 \), \( \frac{\partial l^*}{\partial \tau_l} < 0 \), and \( \frac{\partial l^*}{\partial \tau_k} < 0 \). The introduction of \( \tau_x \) affects \( l^* \) through two opposite effects: the negative effect of \( \tau_x \) on \( l^* \) and the positive effects through the decreases in income taxes. The negative effect of \( \tau_x \) on \( l^* \) has been discussed in Section 3. To understand the positive effect through \( \tau_l \): a decrease in \( \tau_l \) raises the marginal cost of leisure (as in (12c)), so the individual would find it optimal to reduce leisure in the steady state by increasing \( l^* \).

The simulation result shows that, in the reform of replacing \( \tau_l \), the positive effect through decreasing \( \tau_l \) dominates the negative effect of \( \tau_x \) on \( l^* \), so \( l^* \) increases with \( \tau_x \), resulting in more labour supply in the economy. The impact of \( \tau_k \) on \( l^* \) can be separated into two effects. The first effect can be examined from (12c) and (12g): smaller \( \tau_k \) encourage the accumulation of \( k^* \) and thus increases the level of \( c^* \). The individual has to decrease \( l^* \) to maintain the MRS between consumption of the numeraire commodities and leisure. The second effect can be viewed from (20): smaller \( \tau_k \) result in a lower labour to capital ratio, which increases the marginal product of labour. To restore marginal product of labour, \( l^* \) is required to increase. With the parameters specified in Section 4.1, the second effect of \( \tau_k \) dominates, so \( l^* \) increases with reduced \( \tau_k \) in general. The simulation result shows that, in the reform of replacing \( \tau_k \), the negative effect of \( \tau_x \) outweighs the positive effect through decreasing \( \tau_k \). Accordingly, \( l^* \) decreases in \( \tau_x \) in the reform of replacing \( \tau_k \), resulting in higher level of leisure.
The full derivatives of $s^*$ with respect to $\tau_x$ in the two tax reforms yield:

$$\frac{ds^*}{d\tau_x} = \frac{\partial s^*}{\partial \tau_x} + \frac{\partial s^*}{\partial \tau_i} d\tau_i$$

(48)

where $\frac{\partial s^*}{\partial \tau_x} > 0$, $\frac{\partial s^*}{\partial \tau_l} > 0$, and $\frac{\partial s^*}{\partial \tau_k} > 0$. As shown in Table 2, increases in $\tau_x$ alone would pose a positive impact on $s^*$ with the benchmark parameter set. To understand the effects through the channels of decreasing income taxes, it should be noted that decreases in either income taxes would encourage the accumulation of $k$ as shown in (11). To maintain $\dot{k} = 0$ in the steady state, the investment must shifts from the goods sector to the health sector in response in both tax reforms. However, the simulation results show that the positive effect of $\tau_x$ dominates. Therefore, $s^*$ (and thus $v^*$) increases with the implementations of both reforms. However, it should be noted that the sign of $\frac{\partial s^*}{\partial \tau_x} > 0$ varies with different parameter sets, so it is possible that the reforms could have different effects on $s^*$ in different scenarios.\(^4\)

It should be noted that $h^*$ is increasing in $\tau_x$ in both cases. To understand the mechanisms behind them, one can take the full derivations of $h^*$ as below:

$$\frac{dh^*}{d\tau_x} = \frac{\partial h^*}{\partial \tau_x} + \frac{\partial h^*}{\partial \tau_i} d\tau_i$$

(49)

where $\frac{\partial h^*}{\partial \tau_l} < 0$ and $\frac{\partial h^*}{\partial \tau_k} < 0$ as shown in (38) and (39). The overall impacts of the tax reforms on $h^*$ are positive. It should be noted that these increases in $h^*$ are not because of the reduced detrimental effects of decreased $x^*$, but because of the indirect effects from the decreased $\tau_l$ and $\tau_k$.

The comparative-static effects on $k^*$ with the two types of tax reforms can be disentangled into two parts as below:

$$\frac{dk^*}{d\tau_x} = \frac{\partial k^*}{\partial \tau_x} + \frac{\partial k^*}{\partial \tau_i} d\tau_i$$

(50)

\(^4\)The effects of the two reforms in different scenarios can be found in Appendix C
where $\frac{\partial k^*}{\partial \tau_l} < 0$, $\frac{\partial k^*}{\partial \tau_l} < 0$, and $\frac{\partial k^*}{\partial \tau_k} < 0$. The implementation of $\tau_x$ should have a negative effect on $k^*$ as it crowds out the resource available for the accumulation of $k$ (as in (11)). To hold the labour to capital ratio constant in the steady state, $k^*$ has to increase (decrease) as $l^*(h^*)\mu$ increases (decreases). Therefore, the comparative-static effect on $k^*$ can be separated into two parts: the effect of the variations in $l^*$ and the effect of the variations in $h^*$. In the tax reform where $\tau_l$ is replaced by $\tau_x$, the individual would find it optimal to raise $l^*$ in response to the increased marginal labour productivity and the marginal cost of leisure (as in (12c)). Following this increase in $l^*$, $k^*$ has to increase in order to fix the labour to capital ratio. Consequently, the tax reform with the adjustments in $\tau_l$ creates positive effect on $k^*$ through the channel of decreasing $\tau_l$. In the reform with adjustments in $\tau_k$, the decreased $\tau_k$ would encourage the individual to accumulate more $k$. Accordingly, the effect through the channel of decreasing $\tau_k$ is positive. Nevertheless, the negative effects are overshadowed by the positive effects through the channels of decreasing income taxes in both cases. Therefore, $k^*$ is increasing in $\tau_x$ with either reform.

With the quantitative results of the two reforms, one can then simulate the effects on welfare in the economy by taking the quantitative results into the utility function (1). We scale up the utility levels in order to attain positive values. The changes in welfare are plotted in Figure 6.
In Figure 6, the solid curve represents the welfare level with the reform of replacing $\tau_l$ with $\tau_x$, and the dashed curve represents the welfare level with the reform of replacing $\tau_k$ with $\tau_x$. The tax reform where $\tau_l$ is replaced by $\tau_x$ results in decreases in both leisure and $x^*$; nevertheless, due to its contribution to the increase in $c^*$, this tax reform still contributes to better welfare in the long run. Figure 6 also indicates that a switch from $\tau_k$ to $\tau_x$ would result in even better welfare in the economy. Compared to the former reform, the replacement of $\tau_k$ with $\tau_x$ not only increase $c^*$ but also leisure in the long run.

5 Conclusion

This paper provides a rigorous theoretical framework to underpin existing results from the epidemiological literature on population health: first, why taxes on unhealthy commodities alone might fail to improve population health, and second, why these taxes are more likely to be beneficial to health when they are coupled with other fiscal instruments. In addition, we also offer insights on how unhealthy commodities taxation affects the econ-
omy and overall welfare. For this purpose, we construct a dynamic general equilibrium two-sector model with endogenous health. The two sectors employed in the model are the goods sector, which produces consumption commodities, and the health sector, which provides the individual with health. Health produces so-called “healthy time”, which not only affects the level of utility by enhancing leisure time, but also allows the individual to have more time available for work. In this model, the individual has to make trade-offs between investments in both sectors, and the utilities from leisure, consumption of numeraire goods, and that of unhealthy commodities. Although unhealthy commodities provide the individual with utility, they pose detrimental effects on health. Intuitively, taxes on unhealthy commodities should directly affect the level of health in the steady state as long as the taxes are effective in reducing the consumption of unhealthy commodities. However, the steady state solutions show that, even though taxes on unhealthy commodities decrease the consumption of unhealthy commodities, the implementation of these taxes does not directly affect the level of health in the long run. The reason is that, as the detrimental effects decrease, the individual would find it beneficial to reduce the investment in the health sector. Nevertheless, with revenue-neutral adjustments of taxes on labour income or those on capital income, the implementation of taxes on unhealthy commodities can improve the level of health through the channel of income effect. In addition, both tax reforms contribute to higher welfare in the long run. The results offer important guidelines to policy makers: the introduction of a tax on unhealthy commodities, for example a “sugar tax”, should always be coupled with a reduction in other tax burdens in order to improve the level of population health and increase overall welfare.
Appendix A. Benchmark Parameters

<table>
<thead>
<tr>
<th>Benchmark parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of capital in the good sector</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Share of capital in the health sector</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Fraction of labour supply</td>
<td>$l$</td>
</tr>
<tr>
<td>Ratio of numeraire goods to output production</td>
<td>$\frac{c}{y}$</td>
</tr>
<tr>
<td>Ratio of unhealthy commodities to output production</td>
<td>$\frac{x}{y}$</td>
</tr>
<tr>
<td>Health expenditure as a fraction of GDP</td>
<td>$\frac{p_{m}^{m}}{y + p_{m}^{m}}$</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Natural depreciation of health</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Average tax rate on capital income</td>
<td>$\tau_k$</td>
</tr>
<tr>
<td>Average tax rate on labour income</td>
<td>$\tau_l$</td>
</tr>
<tr>
<td>Average tax rate on commodities</td>
<td>$\tau_c$ and $\tau_x$</td>
</tr>
</tbody>
</table>

Appendix B. Comparative-Static Effects of the Income Taxes

Figure 7: Comparative-static effects of $\tau_l$
Figure 8: Comparative-static effects of $\tau_k$

Figure 9: Comparative-static effects of taxes on $m$
Appendix C. Tax Reforms with different Parameter Sets

Figure 10: Tax reform with the adjustment in $\tau_l$ in different scenarios

Figure 11: Tax reform with the adjustment in $\tau_k$ in different scenarios
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