Durham University Business School

Department of Economics

Supplementary appendix to "Information Aggregation Under Ambiguity: Theory and Experimental Evidence"

Spyros Galanis, Christos A. Ioannou, Stelios Kotronis

Working Paper No. 5, 2023

Durham Business School working papers series ISSN 1749-3641

Department Economics
Durham University Business School
Mill Hill Lane
Durham DH1 3LB, UK
Tel: +44 (0)191 3345200
https://www.durham.ac.uk/business/about/departments/economics/

Information Aggregation Under Ambiguity: Theory and Experimental Evidence

Supplementary Appendix

Spyros Galanis* Christos A. Ioannou† Stelios Kotronis‡

Contents

*Durham University Business School. Email: spyros.galanis@durham.ac.uk.

†Centre d'Économie de la Sorbonne. Email: christos.ioannou@univ-paris1.fr.

‡Durham University Business School. Email: stelioskotronis@gmail.com.

1 Uncertainty Averse Preferences

In this section, we show how the characterization of information aggregation, for both the strategic and non-strategic environments, generalizes to the Uncertainty Averse preferences model of Cerreia-Vioglio et al. (2011), which include the Variational preferences (Maccheroni et al. (2006a,b)) and Smooth Ambiguity (Klibanoff et al. (2005)) models. Variational preferences encompass several interesting subcases, such as the MEU preferences that we analyze in the main paper, the Multiplier preferences of Hansen and Sargent (2001), and the Mean-Variance preferences of Markowitz (1952) and Tobin (1958).

The most interesting result of this exercise is that the set of strongly separable securities remains the same as we move from MEU to the much more general framework of Uncertainty Averse preferences. In fact, they are characterized with the same condition, which depends only on the information structure, as shown in Proposition 5. This is surprising because, when we move from EU to MEU, the set of separable securities is a strict subset of the set of strongly separable securities. We discuss the intuition of this result right after Proposition 5.

To generalize Theorems 1 and 2 of the main paper, we assume that the set of "zero cost" priors is fixed, so that it does not depend on the previous announcement, and convex (Assumptions 1 and 2). Although these assumptions hold for MEU, Variational, and Smooth Ambiguity preferences, they are not true for more general Uncertainty Averse preferences. We explain in Section 1.5 why we cannot dispense with these assumptions.

1.1 Model

Each trader evaluates act $f : \Omega \to \mathbb{R}$ as

$$V(f) = \min_{p \in \Delta(\Omega)} G\left(\int u(f)dp, p\right),$$

where $u : \mathbb{R} \to \mathbb{R}$ is a utility index, $G : T \times \Delta(\Omega) \to (-\infty, \infty]$ is quasiconvex and lower semicontinuous on $T \times \Delta(\Omega)$, $T \subseteq \mathbb{R}$, and $\inf_{p \in \Delta(\Omega)} G(r,p) = r$, for all $r \in T$. We also assume that $G(\cdot, p)$ is extended-value continuous on T for each $p \in \Delta(\Omega)$.[1] Traders are risk neutral, so $u(x) = x$, and $G(\cdot, p)$ is strictly increasing for all $p \in \Delta(\Omega)$. Let $\mathcal{P}$ be the set of all beliefs p with finite cost, so that $G(r,p) \neq \infty$, for all $r \in T$.

We also consider two special cases of Uncertainty Averse preferences. First, Variational

[1] That is, $\lim_{t \to t_0} G(t,p) = G(t_0,p) \in (-\infty, \infty]$ for all $t_0 \in T$ and $p \in \Delta(\Omega)$. For instance, $G(t,p) = \infty$ for all $t \in T$ is continuous in this sense. See Cerreia-Vioglio et al. (2011) for more details.

preferences are represented by

$$V(f) = \min_{p \in \Delta(\Omega)} \left(\int u(f) dp + c(p) \right),$$

where $c : \Delta(\Omega) \to [0, \infty]$ is a grounded, convex and lower semicontinuous function. The equivalent Uncertainty Averse representation is for $G(r, p) = r + c(p)$. We refer to $c(p)$ as the "cost" of prior p. Note that this cost is independent of r. MEU preferences are a special case, as $c(p)$ is either zero or infinity.

Second, Smooth Ambiguity preferences are represented by

$$V(f) = \int_{\Delta(\Omega)} \phi \left(\int u(f) dp \right) d\mu(p),$$

where $\phi : \mathbb{R} \to \mathbb{R}$ is a concave and strictly increasing function, and μ is a countably additive Borel probability measure on $\Delta(\Omega)$. Cerreia-Vioglio et al. (2011) show that the equivalent Uncertainty Averse representation is for $G(r, p) = r + \min_{v \in \Gamma(p)} I_r(v \mid\mid \mu)$, where $I_r(\cdot \mid\mid \mu)$ is a statistical distance, so it is quasiconvex and lower semicontinuous with $I_r(v \mid\mid \mu) \geq 0$ for all v and $I_r(\mu \mid\mid \mu) = 0$, for all r. We refer to $\min_{v \in \Gamma(p)} I_r(v \mid\mid \mu)$ as the cost of p. The set $\Gamma(p)$ contains all second-order beliefs v that are absolutely continuous with respect to μ and reduce to the first-order belief p, so that $p = \int q dv(q)$. Note that, unlike with Variational preferences, the cost $\min_{v \in \Gamma(p)} I_r(v \mid\mid \mu)$ depends on r. However, if μ reduces to the first-order belief p, then the cost of p is $\min_{v \in \Gamma(p)} I_r(v \mid\mid \mu) = I_r(\mu \mid\mid \mu) = 0$, for all r. We further assume that, for all $r \in T$, $I_r(v \mid\mid \mu) > 0$ if $v \neq \mu$, so that there is no other p with zero cost.

For general Uncertainty Averse preferences, let $\mathcal{P}_0^r = \left\{ p \in \Delta(\Omega) : G(r, p) = \inf_{q \in \Delta(\Omega)} G\ (r, q) = r \right\}$ be the set of all p that have "zero cost" given $r \in T$. First, we assume that this set is nonempty and fixed, so it does not depend on r.

Assumption 1. $\mathcal{P}_0^r = \mathcal{P}_0^{r'} \neq \emptyset$ *for all* $r, r' \in T$.

Let $\mathcal{P}_0$ be this set of p that have "zero cost" for all r. Behaviorally, probability p has zero cost if and only if the agent is more ambiguity averse than the agent with EU preferences and p as her probability.[2] Second, we assume that $\mathcal{P}_0$ is convex.

Assumption 2. $\mathcal{P}_0$ *is convex.*

Both assumptions are satisfied by Variational preferences, because the cost function $c(\cdot)$ is convex and grounded, so that $\inf_{p \in \Delta(\Omega)} c(p) = 0$, and independent of r. They are also satisfied by

[2] We thank a referee for pointing out this connection.

Smooth Ambiguity preferences, for the unique p which is the reduced, first-order probability of μ, because we have assumed $I_r(v \mid\mid \mu) = 0$ if and only if $v = \mu$, for all $r \in T$. They are not satisfied more generally, as G is quasiconvex and we may have $G(r,p) = r$ but $G(r',p) > r'$. However, both properties are satisfied if there is a unique zero cost p, so that $G(r,q) = r$ if and only if $q = p$, for all $r \in T$.

Let $\mathcal{E}$ be the collection of events where updating occurs, as defined in the main paper. Preferences are updated given an event $E \in \mathcal{E}$ by specifying a new function G_E. Given event E, the trader evaluates act f using $V_E(f) = \min_{p \in \Delta(\Omega)} G_E\left(\int u(f)dp, p\right)$, where $G_E(\cdot, p) = \infty$ if p has a support that is not a subset of E. Let $c = \{G_E\}_{E \in \mathcal{E}}$ be the collection of all such functions. All the aforementioned assumptions for G are maintained for each G_E. For Variational preferences, G_E specifies a cost function $c_E(\cdot)$, whereas for Smooth Ambiguity preferences, the statistical distance given E is $I_{E,r}(\cdot \mid\mid \mu_E)$, where μ_E is the Bayesian update of μ given E.

In the main paper with MEU preferences, we assume prior-by-prior updating. We generalize this updating rule in the following way. Suppose we are in some period t_k and the agent knows that event $F \in \mathcal{E}$ has occurred. Let $\Pi = \{E_1, \ldots, E_m\} \subseteq \mathcal{E}$ be a partition of $F \subseteq \Phi$, containing the events on which the trader can update in period t_{k+1}. For Variational preferences, we require that $c_F(p) \geq \beta \sum_{E \in \Pi} p(E) c_E(p_E)$, for all $p \in \Delta(F)$. Note that this property is weaker than the following updating rule, which Maccheroni et al. (2006a) show is equivalent to Dynamic Consistency,

$$c_F(p) = \inf_{\{q: q(E) = p(E)\ \forall E \in \Pi\}} \left[\beta \sum_{E \in \Pi} p(E) c_E(p_E) + c_F(q)\right].$$

For Smooth Ambiguity preferences, we require that

$$\min_{v \in \Gamma(p)} I_{F,r}(v \mid\mid \mu_F) \geq \beta \sum_{E \in \Pi} p(E) \min_{v \in \Gamma(p_E)} I_{E,r_E}(v \mid\mid \mu_E),$$

for all $r, r_E \in T$. The updating rules for Variational and Smooth Ambiguity preferences generalise to the following rule for Uncertainty Averse preferences. Note that the updating rule is more complicated because there is no separation between the cost of p and the expected utility given p. Consider any sequence of acts f_m, for each $t_m \geq t_k$.[3] We require, for all $p \in \Delta(F)$, that

[3]Recall that there are n traders, so Trader i's payoffs are in periods t_k, t_{k+n}, t_{k+2n}, and so on.

$$G_F\left(E_p\sum_{m=0}^{\infty}\beta^{nm+1}u(f_{k+n+nm}),p\right)\geq\beta\sum_{E\in\Pi}p(E)G_E\left(E_{p_E}\sum_{m=0}^{\infty}\beta^{nm}u(f_{k+n+nm}),p_E\right).$$

Given G_E, let $\mathcal{P}_E$ and $\mathcal{P}_{0E}$ be the set of probabilities with finite and zero costs, respectively. The updating rule implies that if $p\in\mathcal{P}$ (resp. $p\in\mathcal{P}_0$) and $E\in\mathcal{E}$, then $p_E\in\mathcal{P}_E$ (resp. $p\in\mathcal{P}_{0E}$). We say that $c=\{G_E\}_{E\in\mathcal{E}}$ and the corresponding $\mathcal{P}$ are regular with respect to $\mathcal{E}$ if all $p_1,p_2\in\mathcal{P}$ are mutually absolutely continuous with respect to $\mathcal{E}$.

The following lemma is a generalization of Lemma 1 in the main paper.

Lemma 5. *Let s be a continuous strictly proper scoring rule on $[\underline{y},\overline{y}]$, $a,b\in\mathbb{R}$, and let $z\in[\underline{y},\overline{y}]$ be an announcement. Then,*

- $y^*\equiv\arg\max_{y\in[\underline{y},\overline{y}]}\ \mathit{min}_{p\in\Delta(\Omega)}\ G\left(E_p\big[s(y,X)-s(z,X)\big],p\right)$ *is unique,*
- $y^*=E_p[X]$ *for some (not necessarily unique)* $p\in\arg\min_{p\in\Delta}\ \mathit{max}_{y\in[\underline{y},\overline{y}]}\ G\left(E_p\big[s(y,X)-s(z,X)\big],p\right)$,
- *If $z=E_p[X]$ for some $p\in\Delta(\Omega)$ with $G(0,p)=0$, then the optimal announcement is $y^*=z$.*

Proof. Where convenient, we use the notation $s(y)(\cdot)\equiv s(y,X(\cdot))$. We first show that $\arg\max_{y\in[\underline{y},\overline{y}]}\ \min_{p\in\Delta(\Omega)}\ G\left(E_p\big[s(y)-s(z)\big],p\right)$ does, in fact, exist. From Lemma 48 in Cerreia-Vioglio et al. (2011), $\min_{p\in\Delta(\Omega)}\ G\left(E_p\big[s(y)-s(z)\big],p\right)$ is continuous as a function of y. Because $[\underline{y},\overline{y}]$ is compact, a maximum exists and the set $\arg\max_{y\in[\underline{y},\overline{y}]}\ \min_{p\in\Delta(\Omega)}\ G\left(E_p\big[s(y)-s(z)\big],p\right)$ is not empty. Consider the function $H:[\underline{y},\overline{y}]\times\Delta\longrightarrow\mathbb{R}$ defined by $H(y,p)=G(E_p[s(y)-s(z)],p)$.

Instead of Sion's Minimax Theorem, that we used in the main paper, we will use the more general result of Tuy (2011). Given the main topology τ on $\Delta(\Omega)$, define τ' to be the topology on $\Delta(\Omega)$ whose members are complements of sequentially closed subsets of $\Delta(\Omega)$. Topology τ' is finer that τ. Let $a^*>\eta\equiv\max_{y\in[\underline{y},\overline{y}]}\ \min_{p\in\Delta(\Omega)}\ H(y,p)$. We show that the following properties hold for all $a\in(\eta,a^*)$.

1. For every $p\in\Delta(\Omega)$, the set $\{y\in[\underline{y},\overline{y}]:H(y,p)>a\}$ is connected.
2. For every $y\in[\underline{y},\overline{y}]$, the set $C_a(y)=\{p\in\Delta(\Omega):H(y,p)\leq a\}$ is τ'-closed and τ'-connected.
3. For every $p\in\Delta(\Omega)$, the function $H(\cdot,p)$ is upper semicontinuous.

4. $[\underline{y}, \overline{y}]$ is compact.

Note that, for a fixed p, the expected score $E_p\big[s(y) - s(z)\big]$ is single-peaked as a function of y, and the maximum is reached at $y = E_p[X]$. We therefore have that for each b, the set $\{y \in [\underline{y}, \overline{y}] : E_p\big[s(y) - s(z)\big] > b\}$ is connected. Let $a = G(b, p)$. Because $G(\cdot, p)$ is increasing, the set $\{y \in [\underline{y}, \overline{y}] : H(y, p) > a\}$ is connected. Set $C_a(y)$ is closed because G, and therefore H, is lower semicontinuous. Because τ' is finer than τ, $C_a(y)$ is also τ'-closed. For every $p \in \Delta(\Omega)$, $H(\cdot, p)$ is upper semicontinuous because $G(\cdot, p)$ is extended value continuous. Set $C_a(y)$ is τ'-connected for every $y \in [\underline{y}, \overline{y}]$ because it is convex, from the quasiconvexity of G.

From Theorem 1 in Tuy (2011), we have

$$\min_{p \in \Delta(\Omega)} \max_{y \in [\underline{y}, \overline{y}]} G\left(E_p\big[s(y) - s(z)\big], p\right) = \max_{y \in [\underline{y}, \overline{y}]} \min_{p \in \Delta(\Omega)} G\left(E_p\big[s(y) - s(z)\big], p\right).$$

For a fixed p, because s is a strictly proper scoring rule and $G(\cdot, p)$ is strictly increasing, the unique maximizer of $G\left(E_p\big[s(y) - s(z)\big], p\right)$ over $[\underline{y}, \overline{y}]$ is $y = E_p[X]$. Hence, the maxmin is achieved at $p = p^*$ and $y = E_{p^*}[X]$. This proves the second point.

For the first point, suppose there exist $E_p(X) \neq E_q(X)$ that are optimal announcements, both solving

$$\min_{p_0 \in \Delta(\Omega)} G\left(E_{p_0}\big[s(E_{p_0}(X), X) - s(z, X)\big], p_0\right).$$

Then, $G\left(E_p\big[s(E_p(X), X) - s(z, X)\big], p\right) = G\left(E_q\big[s(E_q(X), X) - s(z, X)\big], q\right) \equiv K$. Take the convex combination $ap + (1-a)q$. From the quasiconvexity of G, we have

$$K \geq G\bigg(aE_p\big[s(E_p(X), X) - s(z, X)\big] + (1-a)E_q\big[s(E_q(X), X) - s(z, X)\big], ap + (1-a)q\bigg) =$$

$$G\bigg(aE_p\big[s(E_p(X), X)\big] + (1-a)E_q\big[s(E_q(X), X)\big] - E_{ap+(1-a)q}s(z, X), ap + (1-a)q\bigg) >$$

$$G\bigg(aE_p\big[s(E_{ap+(1-a)q}(X), X)\big] + (1-a)E_q\big[s(E_{ap+(1-a)q}(X), X)\big] - E_{ap+(1-a)q}s(z, X), ap + (1-a)q\bigg) =$$

$$G\bigg(E_{ap+(1-a)q}\big[s(E_{ap+(1-a)q}(X), X) - s(z, X)\big], ap + (1-a)q\bigg).$$

The strict inequality is implied by the fact that s is strictly proper and $G(\cdot, p)$ is strictly increasing for all p. But then $E_p(X)$ and $E_q(X)$ are not optimal, a contradiction. This proves the first point.

For the third point, because the maxmin is equal to the minimax and the proper scoring rule is optimized when announcing $E_p[X]$, we have that the myopic best response is $E_{p^*}[X]$, where

$$p^* \in \arg\min_{p \in \Delta(\Omega)} G\left(E_p\left[s(E_p[X], X) - s(z, X)\right], p\right).$$

Suppose that $z = E_p[X]$ for some $p \in \Delta(\Omega)$ with $G(0, p) = 0$. By announcing $z = E_p[X]$ when the previous announcement is z, the trader gets a 0 payoff and a utility of $G(0, p) = 0$. Because $E_p\left[s(E_p[X], X) - s(z, X)\right] \geq 0$ and $G(\cdot, p)$ is strictly increasing for all p, we have the result.

□

1.2 Strongly separable securities

Given security X and functions $c = \{G_E\}_{E \in \mathcal{E}}$, let

$$d_c(E, v) = \arg\max_{y \in [\underline{y}, \overline{y}]} \min_{p \in \Delta(E)} G_E\left(E_p\left[s(y, X) - s(v, X)\right], p\right)$$

be the (unique from Lemma 5) myopic announcement that maximizes the trader's current period's utility if her information is E and the previous announcement was v. From Lemma 5, the myopic best response is $E_{p^*}[X]$, where

$$p^* \in \arg\min_{p \in \Delta(E)} G_E\left(E_p\left[s(E_p[X], X) - s(v, X)\right], p\right).$$

Definition 9. *A security X is called not strongly separable under partition structure Π and proper scoring rule s if there exist a regular c with respect to each Π_i, $i = 1, ..., n$, and $v \in \mathbb{R}$ such that:*

(i) $X(\omega) \neq v$ for some $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$,

(ii) $d_c(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$.

Otherwise, it is called strongly separable.

Note that MEU is a special case of Variational preferences, which are a special case of Uncertainty Averse preferences. This implies that if a security is not strongly separable with MEU preferences, it is not strongly separable with Uncertainty Averse preferences. The reason is that if we can find a set of beliefs $\mathcal{P}$ for MEU to satisfy (i) and (ii) in Definition 3 of the main paper, we can trivially find a regular $c = \{G_E\}_{E \in \mathcal{E}}$ with Uncertainty Averse preferences such that $G_E(r, p) = r$ for all $p \in \mathcal{P}$ and all $r \in T$, that satisfy (i) and (ii) in Definition 9. Equivalently, if a security is strongly separable with Uncertainty Averse preferences, it is strongly separable with MEU preferences.

We now show that the characterization of strongly separable securities is the same as the one for MEU preferences. Therefore, even for this larger class of preferences, classifying a security as strongly separable does not depend on which scoring rule we use. Moreover, all the results of Subsection 3.6 in the main paper, where we provide examples of strongly separable securities, apply in this more general framework as well.

Proposition 5. *Security X is strongly separable under partition structure Π if and only if for any $v \in \mathbb{R}$, for any non-empty event $E \subseteq \{\omega \in \Omega : X(\omega) \neq v\}$, there exists Trader i, state $\omega \in E$ and $\lambda \in \mathbb{R}$ such that for all $\omega' \in \Pi_i(\omega) \cap E$,*

$$(X(\omega') - v)\lambda > 0.$$

Proof. Suppose that X is not strongly separable for c and v. Then, from Definition 9 part (ii), we have that $d_c(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$. Lemma 5 implies that for each $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p) = E$, for each $i \in I$, we have $E_p[X(\omega) - v|\Pi_i(\omega)] = 0$, for some $p \in \mathcal{P}$, ignoring without loss of generality states ω' for which $X(\omega') = v$. Because $Supp(p) \subseteq E$, it cannot be that for some Trader i, state $\omega \in E$ and $\lambda \in \mathbb{R}$, $(X(\omega') - v)\lambda > 0$ for all $\omega' \in \Pi_i(\omega) \cap E$. For the converse, note that if X is strongly separable for Uncertainty Averse preferences, it is strongly separable for MEU preferences, hence the result follows from Proposition 2 in the main paper.

□

The result is surprising because, when we move from EU to MEU, the set of separable securities is a strict subset of the set of strongly separable securities. The intuition is that since we can employ sets of beliefs, rather than single beliefs, we have more freedom in finding one set that will render a security not strongly separable, by satisfying conditions (i) and (ii) of Definition 3 in the main paper. In the proof of Proposition 2 in the main paper, the freedom of having a set of beliefs allows us to use the Bayesian update of a possibly different prior, one for each partition cell and each trader. This was not possible with EU preferences.

The same intuition would dictate that we can enlarge the set of not strongly separable securities even further with Uncertainty Averse preferences. However, this is not the case. The reason is that part (ii) of Lemma 1 in the main paper generalizes to Uncertainty Averse preferences (part (ii) of Lemma 5 in this Supplementary Appendix). In particular, the myopic announcement is still the expected value of X, according to one belief with finite cost, as opposed to zero cost with MEU preferences. However, any beliefs that we could employ with finite cost in order to classify a security as not strongly separable with Uncertainty Averse preferences, we can also employ with zero cost and MEU preferences, hence the set of not strongly separable securities does not grow further in this more general framework of Uncertainty Averse preferences.

1.3 Non-Strategic environment

The following theorem characterizes information aggregation with respect to strongly separable securities, when traders are myopic.

Theorem 3. *Fix security X, information structure Π and continuous strictly proper scoring rule s. Information gets aggregated for any regular $\Gamma^M(\Omega, I, \Pi, X, \mathcal{P}, y_0, \underline{y}, \overline{y}, s)$ if and only if X is strongly separable.*

Proof. ($\Leftarrow$) Suppose X is strongly separable. By construction, $\mathcal{F}^0(\omega) \supseteq \mathcal{F}^1(\omega) \supseteq ... \supseteq \mathcal{F}^k(\omega)$. Because Ω is finite, there exists t_k such that $\mathcal{F}^{k'}(\omega) = \mathcal{F}^k(\omega)$ for every $t_{k'} \geq t_k$. We denote this set by $\mathcal{F} \equiv \mathcal{F}^k$. Let

$$A^i_{\omega,z} = \left\{ E_p[X] : G_{\Pi_i(\omega)}\left(r, p\right) = r, \text{ where } r = E_p\left(s(E_p[X], X) - s\big(z, X\big)\right) \right\}$$

be the set of all myopic announcements at ω given beliefs p and previous announcement z, that have zero cost. The set is nonempty and convex because $\inf\limits_{p \in \Delta(\Omega)} G_E\ (r, p) = r$, for all $r \in T$ and $E \in \mathcal{E}$, and G is convex on beliefs with zero cost. Because we have assumed that $G(r, p) = r$ implies $G(r', p) = r'$ for all $r' \in \mathbb{R}$, we have $A^i_{\omega,z} = A^i_{\omega,z'}$, hence we write the set as A^i_ω.

We first show that, as $t_k \to t_\infty$, myopic announcements get arbitrarily close to A^i_ω for all $\omega \in \mathcal{F}$. To simplify the notation, we assume, without loss of generality, that $\mathcal{F} = \Omega$. Hence, we write i's private information at all periods $t > t_k$ as $\Pi_i(\omega)$ instead of $\mathcal{F} \cap \Pi_i(\omega)$.

From Lemma 5, Trader i's period t utility from making the myopic announcement at $\omega \in \Omega$ and announcement z is

$$\min_{p \in \Delta(\Pi_i(\omega))} G_{\Pi_i(\omega)}\bigg(E_p\big(s(E_p[X], X) - s(z, X)\big), p\bigg).$$

Because s is a strictly proper scoring rule, for any p and any z, $r = E_p\big(s(E_p[X], X) - s\big(z, X\big)\big)$ is weakly positive. The reason is that with beliefs p it is always optimal to announce $E_p[X]$ because s is a proper scoring rule and $G_{\Pi_i(\omega)}(r, p)$ is strictly increasing in r. By repeating the previous announcement z, the agent can get 0, so she can do weakly better by announcing $E_p[X]$. If the myopic announcement is outside A^i_ω, then i's utility is strictly positive because $G(r,p) > r$. We now show that if i's announcements are consistently outside of A^i_ω, as $t_k \to t_\infty$, then i's utility is bounded below by a strictly positive number.

Lemma 6. *Take any sequence $\{y_k\}$ of Trader i's announcements at times $\{t_k\}$ that are myopic best responses to the announcements of Trader $i-1$. If each y_k is outside of $[\min A^i_\omega - \epsilon, \max A^i_\omega + \epsilon]$, for some $\epsilon > 0$, then i's period payoff is bounded below by a strictly positive number.*

Proof. Take a sequence of i's myopic announcements $y_k \to y$ at times $\{t_k\}$ that are always outside of $[\min A^i_\omega - \epsilon, \max A^i_\omega + \epsilon]$, and collect the sequence $p_k \to p$ of the corresponding beliefs p_k such that $y_k = E_{p_k}[X]$. If each y_k is outside $[\min A^i_\omega - \epsilon, \max A^i_\omega + \epsilon]$, then $E_{p_k}[X]$ converges to some $E_p[X] \notin A^i_\omega$. Because $E_p\left(s(E_p[X],X) - s(z,X)\right)$ is always weakly positive, we have that $G_{\Pi_i(\omega)}(E_p\left(s(E_p[X],X) - s(z,X)\right), p) = \eta > 0$. From the lower semicontinuity of G we have that

$$\liminf_{p_k \to p} G_{\Pi_i(\omega)}\left(E_{p_k}\left(s\big(E_{p_k}[X],X\big) - s(y_{k-1},X)\right), p_k\right) \geq$$

$$G_{\Pi_i(\omega)}\left(E_p(s(E_p[X],X) - s(z,X)), p\right) = \eta > 0.$$

□

We now show that it is not possible that i's period payoff is bounded below by a strictly positive number η. Suppose not. Fix probability $q \in \Delta(\Omega)$ such that $G(r,q) = r$, for all $r \in T$. Such a q exists by Assumption 1. Because we minimize over all $p \in \Delta(\Pi_i(\omega))$, we have that $G_{\Pi_i(\omega)}\left(E_p\left(s(E_p[X],X) - s(z,X)\right), p\right) > \eta > 0$ for all such p. Therefore, this is true also for the Bayesian update of q, given $\Pi_i(\omega)$, denoted $q_{\Pi_i(\omega)}$. We therefore have $E_{q_{\Pi_i(\omega)}}\left(s(E_{q_{\Pi_i(\omega)}}[X],X) - s(z,X)\right) > \eta > 0$. Repeating the same procedure for all $\omega \in \mathcal{F}$ and corresponding $\Pi_i(\omega)$, then integrating using that q, we have that $E_q\left(s(E_q[X],X) - s(z,X)\right) > \eta^* > 0$, where the expectation is over $\mathcal{F}$ and η^* is the expectation over all η.[4]

Denote $E_q\left(s(E_q[X],X) - s(z,X)\right)$ as the expected score $\Psi_k = \bar{s}_k - \bar{s}_{k-1}$ of the trader who makes the announcement at t_k. Note that Ψ_k is weakly positive. Additionally, it is bounded below by η^* if Trader i at t_k makes a myopic announcement outside of $[\min A^i_\omega - \epsilon, \max A^i_\omega + \epsilon]$, for some $\omega \in \mathcal{F}$, where $\epsilon > 0$ is uniform across t_k.

[4]Note that for all other states ω' the payoff is weakly positive. Also, recall that we have assumed, without loss of generality, that $\mathcal{F} = \Omega$.

The limit $\lim_{K\to\infty}\sum_{k=1}^{K}\Psi_k$ must be infinite because each Ψ_k is weakly positive and an infinite number of them is greater than η^*. However, for any K, we have

$$\begin{aligned}\sum_{k=1}^{K}\Psi_k &= (\overline{s}_1-\overline{s}_0)+\\ &+(\overline{s}_2-\overline{s}_1)+\\ &+\quad\vdots\\ &+(\overline{s}_K-\overline{s}_{K-1})\\ &=(\overline{s}_K-\overline{s}_0)\\ &\le 2M,\end{aligned}$$

where $M = \max_{y\in[\underline{y},\overline{y}],\omega\in\Omega}|s(y,X(\omega))|$. This is a contradiction, hence each agent i makes myopic announcements that are arbitrarily close to A^i_ω as $t_k \to t_\infty$ for all $\omega \in \mathcal{F}$. By applying similar arguments to the proof of Theorem 1 in the main paper for the MEU case, where the myopic announcements are always inside A^i_ω, we establish that there is information aggregation.

($\Rightarrow$) The proof is identical to that for the MEU case. Suppose that for any regular Γ^M, information gets aggregated so that $y_k(\omega) = d_c(\Pi_{a(t_k)}(\omega)\cap\mathcal{F}^{k-1}(\omega), y_{k-1}) \longrightarrow X(\omega)$ for every $\omega \in \bigcup_{p\in\mathcal{P}} Supp(p)$. We show that, for any regular c and $v\in\mathbb{R}$, if (ii) in Definition 9 is satisfied, then, (i) is violated.

Suppose there exist regular c and $v \in \mathbb{R}$ such that $d_c(\Pi_i(\omega), v) = v$ for all $i = 1, ..., n$ and $\omega \in \bigcup_{p\in\mathcal{P}} Supp(p)$. Consider regular $\Gamma^M(\Omega, I, \Pi, X, c, y_0, [\underline{y}, \overline{y}], s)$ with initial announcement $y_0 = v$. Then, the predictions $y_{t_k}(\omega)$, $k = 0, 1, ...,$ are equal to v for all $\omega \in \bigcup_{p\in\mathcal{P}} Supp(p)$. If we have $X(\omega) \neq v$ for some $\omega \in \bigcup_{p\in\mathcal{P}} Supp(p)$, then, at ω all traders agree on v, which is the wrong value of the security. This implies that there is no information aggregation, a contradiction. Hence, condition (i) in Definition 9 is violated and X is strongly separable.

$\square$

1.4 Strategic environment

The details of the strategic environment are described in the main paper. The main difference, here, is that the continuation value of each trader is determined by $c = \{G_E\}_{E\in\mathcal{E}}$. Recall that the state space is $\Phi = \Omega \times [0,1]^{\mathbb{N}}$.

Definition 10. *The continuation payoff of player $a_k = i$ at time t_k and state ϕ, given strategy profile σ, history H^{k-1} and system of beliefs $\mathscr{P}$ is*

$$V(H^{k-1}, \phi, \sigma, \mathscr{P}) =$$

$$\min_{p \in \mathcal{P}(H^{k-1}, \phi)} G_{\Pi_i^k(\phi)} \left[E_p \sum_{m=0}^{\infty} \beta^{nm} \left(s\Big(y_{k+nm}(\sigma, \phi | H^{k-1}), X(\phi)\Big) - s\Big(y_{k+nm-1}(\sigma, \phi | H^{k-1}), X(\phi)\Big) \right), p \right].$$

In the main paper, we characterize information aggregation in terms of Revision-Proof equilibria (Theorem 2). The proof for Uncertainty Averse preferences is similar. The main difference is that we need to account for p with positive cost, so that $G(r, p) > r$.

Before proving Theorem 4, we state the following auxiliary result, which shows that a trader's continuation value is always greater than her one-period payoff.

Proposition 6. *In a Revision-Proof equilibrium, the continuation value for Trader i who plays at t_k is at least as much as her utility from the one-period payoff from playing the myopic best response.*

Proof. We construct a deviation strategy that guarantees a continuation value at least as much as that of the one-period payoff from playing the myopic strategy. We will show that for each t_k, the continuation payoff of Trader i who makes the announcement is weakly more than χ_k, her one-period payoff from playing the myopic strategy at t_k.

We define a deviation strategy $\sigma = (\sigma_i, \sigma_{-i}^*)$, where all traders $j \neq i$ follow the equilibrium strategy σ^* and σ_i is identical to σ_i^* up to time t_{k-1}. At t_k, σ_i specifies that Trader i plays the myopic best response. Given that i deviates and all other traders stick to the equilibrium strategy σ^*, let $H^1, ..., H^m$ be the possible paths of announcements by all other traders $j \neq i$ from t_k to t_{k+n-1}, together with the common history of announcements up to t_{k-1}. They are finitely many because we consider mixing over finite actions. At t_{k+n}, σ_i specifies that:

(*a*) If $V(H^m, \phi, \sigma, \mathscr{P}) \geq 0$ by playing what σ_i^* prescribes at H^m, then σ_i coincides with σ^* in every succeeding information set,

(*b*) If $V(H^m, \phi, \sigma, \mathscr{P}) < 0$, then σ_i repeats the previous trader's prediction.

If (a) occurs, then σ_i coincides with σ^* in every succeeding information set, so Trader i follows the recommendation of σ_i. If (b) occurs, then Trader i repeats the previous announcement and in every succeeding information set, σ_i is determined using the two cases (a) and (b). For every other information set not specified by the above procedure, σ_i is identical to σ_i^*.

We now show that at (b), Trader i will follow the recommendation to repeat the previous announcement and get a period payoff of zero. This is true if her continuation value, excluding her current period payoff, is weakly positive, as long as all future selves follow σ. We now show that this is true at all $t \geq t_k$. We show this for $t = t_k$, without loss of generality, and note that, from i's perspective, there are two types of subsequent paths, given that everyone follows σ. The first type is a path that specifies some zero payoffs initially, and at some time $t > t_{k+n}$ the continuation value of i's future self is weakly positive by playing σ^* onwards. The second type is a path where the future selves just repeat the previous announcement because from σ^* they would get a negative continuation value, hence the payoffs along this path are zero always. This means that all paths have a weakly positive continuation value at some time $t > t_k$, and the previous payoffs between t_k and t are zero. Hence, it is without loss of generality to assume that the future selves at period t_{k+n} and at each path, compute weakly positive continuation value. However, because of Dynamic Inconsistency the continuation value at some path at t_{k+n} may be evaluated at a different prior than the one that Trader i uses at t_k to evaluate her own continuation value. The collection of all paths generates a partition Π of state space Φ and σ generates a sequence of acts f_m, for each $t > t_k$. We therefore have, for each $E \in \Pi$, and from the perspective of the future selves in time t_{k+n}, that

$$0 \leq \min_{p \in \mathcal{P}} G_E\left(E_{p_E} \sum_{m=0}^{\infty} \beta^{nm} u(f_{k+n+nm}), p_E\right) = G_E\left(E_{q_E} \sum_{m=0}^{\infty} \beta^{nm} u(f_{k+n+nm}), q_E\right).$$

At every partition cell E, the future self at t_{k+n} chooses a potentially different belief q_E. Let p be the belief that Trader i uses at t_k to compute her continuation value. We then have that

$$0 \leq G_E\left(E_{q_E} \sum_{m=0}^{\infty} \beta^{nm} u(f_{k+n+nm}), q_E\right) \leq G_E\left(E_{p_E} \sum_{m=0}^{\infty} \beta^{nm} u(f_{k+n+nm}), p_E\right).$$

By multiplying with β and $p(E)$, and adding over all $E \in \Pi$, we have

$$0 \leq \beta \sum_{E \in \Pi} p(E) G_E\left(E_{p_E} \sum_{m=0}^{\infty} \beta^{nm} u(f_{k+n+nm}), p_E\right) \leq G_F\left(E_p \sum_{m=0}^{\infty} \beta^{nm+1} u(f_{k+n+nm}), p\right),$$

where the second inequality is due to the updating rule we have assumed in Section 1.1. This shows that the continuation value at any $t \geq t_k$ is weakly positive if Trader i repeats the previous announcement at t and gets a period payoff of zero. Therefore, she will always follow the recommendation at (b), if by sticking to σ^* her continuation value is strictly negative.

At t_k, Trader i plays her myopic best response and gets a period payoff of χ_k, which is weakly positive, and evaluated at some q so that her period utility is $\min_{q\in\mathcal{P}} G_F\Big(\chi_k, q\Big)$. Because $G(\cdot, p)$ is strictly increasing in the first argument for all p and $G_F(0, p) = 0$, we have that $E_p \sum_{m=0}^{\infty} \beta^{nm+1} u(f_{k+n+nm}) \geq 0$. Because $G_F(\cdot, p)$ is strictly increasing in the first argument for all p, we have $G_F\Bigg(\chi_k + E_p \sum_{m=0}^{\infty} \beta^{nm+1} u(f_{k+n+nm}), p\Bigg) \geq G_F\Big(\chi_k, p\Big) \geq \min_{q\in\mathcal{P}} G_F\Big(\chi_k, q\Big)$. Hence, her continuation value is always weakly greater than her period payoff by playing the myopic best response.

□

Theorem 4. *Fix information structure* Π *and bounds* $[\underline{y}, \overline{y}]$.

- (i) *If security* X *is strongly separable under* Π*, then for any* Γ^S *and any Revision-Proof equilibrium, information aggregates.*
- (ii) *If security* X *is not strongly separable under* Π*, then there exist game* Γ^S *and a Revision-Proof equilibrium such that information does not get aggregated.*

Proof of Theorem 4. The proof follows closely the proof for the MEU case. The only changes are in steps 1 and 4, so we omit steps 2 and 3 of the proof of Theorem 2 in the main paper.

Step 1: We show that if the security is strongly separable and its value is not constant for each state in the support of the set of beliefs, at least one trader can achieve a strictly positive payoff at some state and a weakly positive payoff at all other states whatever the previous announcement.

Let $\mathcal{P}_k$ be the beliefs over Ω of an outside observer who hears the announcements up to t_{k-1} and updates the initial set of beliefs $\mathcal{P}$ given the equilibrium strategies, but has no private information about Ω. Let $Supp(\mathcal{P}_k)$ be the union of the supports of all $p \in \mathcal{P}_k$. For each $\omega \in Supp(\mathcal{P}_k)$ and $i \in I$, let $E = Supp(\mathcal{P}_k) \cap \Pi_i(\omega)$ and define

$$A_{\omega}^{ik} \equiv \Big\{E_p[X] : p \in \mathcal{P}_k, G_E\left(r, p\right) = r, \text{ where } r = E_p(s(E_p[X], X) - s(z, X))\Big\}$$

to be the set of all myopic announcements of Trader i at ω. Note that A_{ω}^{ik} does not depend on the previous announcement z, because of Assumption 1 that the zero cost probabilities are independent of r.

This set is non-empty, convex from Assumption 2, and compact. Let $\min A_{\omega}^{ik}$ ($\max A_{\omega}^{ik}$) be the minimum (maximum) value. We first show that, in any equilibrium, the announcement of Trader i gets arbitrarily close to the announcement of Trader $i-1$ and to A_{ω}^{ik}, for all $\omega \in Supp(\mathcal{P}_k)$, as $t_k \to t_\infty$. Note that $A^{ik} = \bigcap_{\omega \in Supp(\mathcal{P}_k)} A_{\omega}^{ik}$ cannot be empty, otherwise the outside observer would understand that some state ω is not true, because the announcements do not get arbitrarily close to A_{ω}^{ik} as $t_k \to t_\infty$. Hence, the announcements get arbitrarily close to A^{ik} as well.

Lemma 7. *For any $\epsilon > 0$ and Trader i, there is period t' such that for all $t_k > t'$ where i makes an announcement, $|y_k - y_{k-1}| < \epsilon$ and $y_k \in [\min A_{\omega}^{ik} - \epsilon, \max A_{\omega}^{ik} + \epsilon]$, for all $\omega \in Supp(\mathcal{P}_k)$.*

Proof. We first show that, for any $\epsilon > 0$, if Trader $i-1$'s announcement z is outside $[\min A_{\omega}^{ik} - \epsilon, \max A_{\omega}^{ik} + \epsilon]$, for some state ω and time t_k, then i's expected payoff from playing her myopic best response is greater than some $\chi_k > 0$, and that χ_k cannot converge to 0 as $t_k \to t_\infty$. There are two cases. First, i's myopic announcement is outside of $[\min A_{\omega}^{ik} - \epsilon', \max A_{\omega}^{ik} + \epsilon']$ for some $\epsilon' > 0$ and some subsequence $\{t_k\}$ of periods. Then, Lemma 6 shows that, irrespective of the announcements of $i-1$, i's period payoff is bounded below by a strictly positive number. Second, i's myopic announcement is always inside $[\min A_{\omega}^{ik}, \max A_{\omega}^{ik}]$ after some t'. This is the MEU case, which is covered in Lemma 3 of the main paper.

We next show that χ_k cannot converge to 0 as $t_k \to t_\infty$. From Proposition 6, Trader i's continuation payoff in equilibrium must be weakly higher than her one-period payoff χ_k. This implies that if Trader $i-1$ makes announcements outside of $[\min A_{\omega}^{ik} - \epsilon, \max A_{\omega}^{ik} + \epsilon]$ for infinitely many t_k, then i's expected continuation payoff (which is greater than χ_k) does not converge to zero. We now show that this is impossible.

Suppose not. Then, the expected continuation payoff for i is bounded below by a positive number. For all other traders it is weakly positive, again using Proposition 6, and because their one-period payoff is always weakly positive. Because the continuation payoff is minimized over all beliefs in $\mathcal{P}_k$, we can pick a prior p with zero cost, take its Bayesian update at the decision node at time t_k and define Ψ_k to be the sum of all traders' expected continuation payoffs (given that p) at t_k, divided by β^k,

$$\Psi_k = (\overline{s}_k - \overline{s}_{k-1}) + \beta(\overline{s}_{k+1} - \overline{s}_k) + \beta^2(\overline{s}_{k+2} - \overline{s}_{k+1}) + \dots$$

Recall that if $p \in \mathcal{P}_0$, all of its Bayesian updates belong to $\mathcal{P}_0$ and therefore have zero cost as well. The $\overline{s}_k$ is the expected score of prediction y_k, where the expectation is over all ϕ, given some $p \in \mathcal{P}_k$ and the moves of players according to the mixed equilibrium.

For any K, we have

$$
\begin{aligned}
\sum_{k=1}^{K} \Psi_k &= (\overline{s}_1 - \overline{s}_0) + \beta(\overline{s}_2 - \overline{s}_1) + \beta^2(\overline{s}_3 - \overline{s}_2) + \dots \\
&+ (\overline{s}_2 - \overline{s}_1) + \beta(\overline{s}_3 - \overline{s}_2) + \beta^2(\overline{s}_4 - \overline{s}_3) + \dots \\
&+ \quad \vdots \\
&+ (\overline{s}_K - \overline{s}_{K-1}) + \beta(\overline{s}_{K+1} - \overline{s}_K) + \beta^2(\overline{s}_{K+2} - \overline{s}_{K+1}) + \dots \\
&= (\overline{s}_K - \overline{s}_0) + \beta(\overline{s}_{K+1} - \overline{s}_1) + \beta^2(\overline{s}_{K+2} - \overline{s}_2) + \dots \\
&\leq 2M/(1-\beta),
\end{aligned}
$$

where $M = \underset{y \in [\underline{y}, \overline{y}], \omega \in \Omega}{max} |s(y, X(\omega))|$. But this contradicts the fact that i's expected continuation payoff is bounded below by a positive number. We then have that, in equilibrium, Trader $i-1$ makes announcements that are arbitrarily close to A^{ik}_{ω}, for each $\omega \in Supp(\mathcal{P}_k)$, hence arbitrarily close to A^{ik}. Note that, because $\sum_{k=1}^{K} \Psi_k$ is bounded above by a positive number for any K, and each Ψ_k is weakly positive, we have that $\lim_{K \to \infty} \sum_{k=1}^{K} \Psi_k = \chi_0$ for some finite χ_0.

We finally show that, given that $i-1$ announces arbitrarily close to A^{ik}, the announcement of i gets arbitrarily close to the announcement of $i-1$ in equilibrium, and therefore the announcements of i get arbitrarily close to A^{ik}. Suppose not, so that $|y_k - y_{k-1}| > \epsilon$ for a fixed ϵ and for infinitely many t_k, where i makes an announcement. Suppose that in every t_k, where i makes an announcement, we evaluate i's period payoff at t_k using $p_k \in \mathcal{P}^t_{\Pi_i(\omega)}$, such that $E_{p_k}[X] = y_{k-1}$ if $y_{k-1} \in A^{ik}_{\omega}$, $E_{p_k}[X] = \min A^{ik}_{\omega}$ if $y_{k-1} < \min A^{ik}_{\omega}$ (but arbitrarily close to it) or $E_{p_k}[X] = \max A^{ik}_{\omega}$ if $y_{k-1} > \max A^{ik}_{\omega}$ (but arbitrarily close to it).[5] In all cases and since $i-1$'s announcement is arbitrarily close to A^{ik}_{ω}, we have that i's period payoff, $E_{p_k}\Big(s\big(y_k, X\big) - s\big(y_{k-1}, X\big)\Big)$, is strictly negative. As scoring rules are order sensitive, the period payoff will also be strictly negative if i's announcement is exactly ϵ away from the announcement of $i-1$. By collecting these p_k for all such t_k, we have that $E_{p_k}\Big(s\big(E_{p_k}[X] + \epsilon, X\big) - s\big(E_{p_k}[X], X\big)\Big) < 0$, where $s\big(y_{k-1}, X\big)$ is arbitrarily close to $s\big(E_{p_k}[X], X\big)$ by continuity.[6]

[5]Note that if the previous announcement z is inside A^{ik} and therefore inside each A^{ik}_{ω}, then there exists $p \in \mathcal{P}_0$ with $E_p[X]$ and zero cost. But then, the third point of Lemma 5 implies that i's myopic response is to repeat it. Therefore, we are back to the MEU case, where all priors have zero cost.

[6]We assume, without loss of generality, that i's announcement is always ϵ-higher than the previous

Since the set of all beliefs is compact, there is a converging sequence $\{p_k\}$ of beliefs. If $\lim_{p_k \to p} E_{p_k}\bigg(s\big(E_{p_k}[X]+\epsilon, X\big) - s\big(E_{p_k}[X], X\big)\bigg) = 0$, the continuity of the scoring rule implies that $E_p\bigg(s\big(E_p[X]+\epsilon, X\big)\bigg) = E_p\bigg(s\big(E_p[X], X\big)\bigg)$ so that both announcements $E_p[X]+\epsilon$ and $E_p[X]$ are optimal given p, contradicting that s is a strictly proper scoring rule. If $\lim_{p_k \to p} E_{p_k}\bigg(s\big(E_{p_k}[X]+\epsilon, X\big) - s\big(E_{p_k}[X], X\big)\bigg) < 0$, i's period payoff given some beliefs p_k is bounded above by a strictly negative number. But this is the first element of some Ψ_k. We have already shown that $\sum_{k=1}^{K} \Psi_k$ is bounded above by a positive number for each K and each Ψ_k is weakly positive because a trader can always repeat the previous announcements as shown in Proposition 6. Therefore, we have that $\lim_{k \to \infty} \Psi_k = 0$, which contradicts that the first term can be bounded above by a negative number. Since this is true for all states in $Supp(\mathcal{P}_k)$, the above statements are also true for A^{ik} and the result follows. That is, given that $i-1$ announces arbitrarily close to A^{ik}, the announcement of i gets arbitrarily close to the announcement of $i-1$ in equilibrium, and therefore the announcements of i get arbitrarily close to A^{ik}.

□

Given an equilibrium, the updating of beliefs $\mathcal{P}_k$ may never stop for sufficiently high t_k as traders play their mixed strategies and do prior-by-prior updating. Let $\mathcal{P}$ be a set of limit beliefs of this sequence $\{\mathcal{P}_k\}$ with some probability. Let $\mathcal{D}$ be the collection of these sets of limit beliefs that describe some uncertainty about the value of the security. That is, for each $\mathcal{P} \in \mathcal{D}$, there exist $\omega, \omega' \in \bigcup_{p \in \mathcal{P}} Supp(p)$ such that $X(\omega) \neq X(\omega')$.

From Lemma 5, we know that given beliefs $\mathcal{P} \in \mathcal{D}$ and at any state $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$, each Trader j can achieve a weakly positive payoff by making the myopic announcement $d_c(E, z)$, where z is the previous announcement.

Generalizing the notion of Ostrovsky (2012), we define the instant opportunity of Trader i, given regular c and previous announcement z, to be

$$\min_{q \in \mathcal{P}} G\left[\sum_{\omega \in \Omega} q(\omega)\left[\min_{p \in \mathcal{P}_{\Pi_i(\omega)}} G_{\Pi_i(\omega)}\left[E_p\bigg(s\big(E_p[X], z\big), X\big) - s\big(z, X\big)\bigg), p\right]\right], q\right]$$

Note that at each partition cell $\Pi_i(\omega)$, Trader i chooses a possibly different $p \in \mathcal{P}_{\Pi_i(\omega)}$ that minimizes her expected utility. The instant opportunity is the ex ante (minimal over $\mathcal{P}$) utility aggregating over all partition cells.

---

announcement. Similar arguments can be employed if it was always ϵ-lower or it was alternating as we can always get a subsequence of p_k for which i's announcement is always higher (or lower).

The following lemma shows that if the security X is strongly separable and beliefs $\mathcal{P} \in \mathcal{D}$ describe some uncertainty about X, then the instant opportunity of some Trader i is strictly positive, irrespective of what the previous announcement is.

Lemma 8. *If security X is strongly separable, then for every $\mathcal{P} \in \mathcal{D}$ there exist $\chi > 0$ and $i \in I$ such that, for every $z \in \mathbb{R}$, the instant opportunity of i given $\mathcal{P}$ and z is greater than χ.*

Proof. Note that the expression for the instant opportunity inside the brackets,

$$\min_{p \in \mathcal{P}_{\Pi_i(\omega)}} G_{\Pi_i(\omega)} \left[E_p \Big(s\big(E_p[X], z\big), X\big) - s\big(z, X\big) \Big), p \right], \tag{1}$$

is i's expected payoff given $\Pi_i(\omega)$, when making the myopic announcement and the previous announcement is z. From Lemma 5, this is weakly positive for all $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p)$. Moreover, because $\mathcal{P}$ is regular, each $p \in \mathcal{P}$ assigns positive probability to each $\Pi_i(\omega)$, where $\omega \in \bigcup_{p \in \mathcal{P}} Supp(p) = E$. Therefore, we only need to show that there exists some trader $i \in I$, such that for any z, there is some $\Pi_i(\omega)$ for which the expression in (1) is above a strictly positive lower bound. Note that the lower bound must be the same for all z.

For each ω and $i \in I$, define

$$A^i_\omega = \left\{ E_p[X] : G_{\Pi_i(\omega)}\,(r, p) = r, \text{ where } r = E_p\left(s\big(E_p[X], X\big) - s\big(z, X\big) \right) \right\}$$

and let $\min A^i_\omega$ ($\max A^i_\omega$) be the minimum (maximum) value. Recall that, as we have explained before, A^i_ω is nonempty, convex, compact, and independent of the previous announcement z. Let $A^i = \bigcap_{\omega \in \mathcal{F}} A^i_\omega$. There are three cases.

Case 1: For some i, $A^i = \emptyset$.

There are two subcases. First, the myopic best response, $E_p[X]$, given the previous announcement z, is for p with positive cost, for some ω and Trader i, so that $G_{\Pi_i(\omega)}(r, p) > r$, where r is i's payoff from the announcement. As we show in the proof of Lemma 6, her period utility is strictly positive, hence she has an instant opportunity. Second, the myopic best response is for p with $G_{\Pi_i(\omega)}(r, p) = r$. Then, as we show in the MEU environment, Case 1 of Lemma 4 in the main paper, there is an instant opportunity, irrespective of the previous announcement.

Case 2: $A^i \neq \emptyset$ for all $i \in I$ and $\bigcap_{j \in I} A^j \neq \emptyset$.

This is the same as Case 2 in the proof of Theorem 1 in the main paper. There are two subcases. First, in all states that are considered possible, security X pays the same. This is impossible, because we have assumed that there is uncertainty about X given $\mathcal{P}$. Second, there is uncertainty about X. As we show in Case 2 in the proof of Theorem 1 in the main paper, this implies that X is not strongly separable, a contradiction.

Case 3: $A^i \neq \emptyset$ for all $i \in I$ but $\bigcap_{j\in I} A^j = \emptyset$.

We will show that this case is impossible. Lemma 7 shows that in any set of beliefs $\mathcal{P}$ that can arise in equilibrium after a sufficiently large t_k, i's announcements get arbitrarily close to A^i. Moreover, Trader i's announcements get arbitrarily close to the announcements of $i-1$, which get arbitrarily close to A^{i-1}. At the limit set of beliefs, $\mathcal{P}$, we have that $A^{i-1} \cap A^i \neq \emptyset$ for each $i \in I$. Continuing inductively over all traders, we have that $\bigcap_{j\in I} A^j \neq \emptyset$, a contradiction.

$\square$

Steps 2 and 3 are identical to those for the MEU environment so we omit them.

Step 4: This step concludes the proof by showing that the presence of a "non-vanishing arbitrage opportunity" is impossible in equilibrium.

Let $\mathcal{P}(H^{k-1})$ be the set of updated beliefs for the outside observer at time t_k, given the mixed equilibrium, the set of prior beliefs $\mathcal{P}$ and history H^{k-1}. Note that with mixed strategies, H^{k-1} occurs with some probability. Moreover, because the equilibrium profile may consist of mixed strategies, $\mathcal{P}(H^k)$ may not be the same as $\mathcal{P}(H^{k-1})$, however for big enough t_k, they will have the same support on the state space Ω as it is finite. Consider such a big enough t_k.

Fix t_k, history H^{k-1} and suppose i makes an announcement. Her continuation payoff given history H^{k-1} and state ϕ is

$$V(H^{k-1}, \phi) = \min_{p \in \mathcal{P}(H^{k-1},\phi)} G_{(H^{k-1},\phi)}\left(E_p \sum_{m=0}^{\infty} \beta^{nm} \left(s_{k+nm}(\phi') - s_{k+nm-1}(\phi')\right), p\right),$$

where $s_{k+nm}(\phi')$ is the score at state ϕ' and time t_{k+nm}.

Using Proposition 6, her continuation payoff $V(H^{k-1}, \phi)$ is greater than the one-period payoff from playing the myopic strategy at t_k. Because this is true for all states $\phi \in \bigcup_{p\in\mathcal{P}(H^{k-1})} Supp(p)$ that the outside observer considers possible at t_k, given history H^{k-1}, we have that $\min_{p\in\mathcal{P}(H^{k-1})} G_{H^{k-1}}\left(E_p V(H^{k-1}, \phi), p\right)$ is greater than i's instant opportunity given beliefs $\mathcal{P}(H^{k-1})$.

Again using Proposition 6, the continuation payoff at t_k of each Trader $j \neq i$, who announces at t_k, is weakly positive at each state ϕ and history H^{k-1}. Since this is true for all states $\phi \in \bigcup_{p\in\mathcal{P}(H^k)} Supp(p)$, we have that $\min_{p\in\mathcal{P}(H^{k-1})} G_{H^{k-1}}\Big(E_pV(H^{k-1},\phi),p\Big) \geq 0$.

Since $\min_{p\in\mathcal{P}(H^{k-1})} G_{H^{k-1}}\Big(E_pV(H^{k-1},\phi),p\Big)$ is weakly positive for each $i \in I$, we have that $\sum_{i\in I} G_{H^{k-1}}\Big(E_pV(H^{k-1},\phi),p\Big)$ is weakly positive for any $p \in \mathcal{P}(H^{k-1})$. Moreover, it is strictly positive if i's instant opportunity is strictly positive given $\mathcal{P}(H^{k-1})$. Since this is true for all $p \in \mathcal{P}(H^{k-1})$, fix prior $q \in \mathcal{P}_0$ with zero cost. By assumption, all of its Bayesian updates will also have zero cost. For any previous announcement and considering the (unique) probability over histories H^{k-1} that can arise at t_k, generated by the (possibly) mixed equilibrium, we can let Ψ_k be the sum of all players' expected continuation payoffs at t_k, divided by β^k as

$$\Psi_k = (\bar{s}_k - \bar{s}_{k-1}) + \beta(\bar{s}_{k+1} - \bar{s}_k) + \beta^2(\bar{s}_{k+2} - \bar{s}_{k+1}) + \dots$$

The $\bar{s}_k$ is the expected score of prediction y_k, where the expectation is over all ϕ, given the fixed $q \in \mathcal{P}$ with zero cost and the moves of players according to the mixed equilibrium. We keep $q \in \mathcal{P}$ constant for all t_k. We then have that Ψ_k is weakly positive. Additionally, it is strictly positive if i's expected instant opportunity is strictly positive and it is i's turn to make an announcement. That is, with some probability, some history H^{k-1} occurs and i's instant opportunity is strictly positive.

The last step is identical to that of Ostrovsky (2012) because all Ψ_k are calculated using the same prior $q \in \mathcal{P}$. The proof of Lemma 7 shows that $\lim_{K\to\infty} \sum_{k=1}^{K} \Psi_k = \chi_0$ for some finite χ_0. From Step 3 (omitted in this proof because it is identical to the MEU case), this limit must be infinite because each Ψ_k is weakly positive and an infinite number of them is greater than η^*. Hence, both cases of Step 3 are impossible and y_k must converge to the intrinsic value of security X.

Part (ii) is identical to the one for the MEU environment so we omit it.

□

1.5 Fixed set of zero cost priors

In order to prove Theorems 3 and 4, we have maintained Assumption 1, that the set $\mathcal{P}_0$ of zero cost priors is fixed, so that if $G(r,p) = r$ for some $r \in T$, then $G(r',p) = r'$ for all $r' \in T$. This property is true for Variational preferences and it is also true for Smooth Ambiguity preferences, as long as $\mathcal{P}_0$ is always a singleton. However, it is not true for more general Uncertainty Averse preferences. We now argue why we cannot dispense with this assumption.

In the proofs for both the myopic and the strategic environments, we show that, as $t_k \to t_\infty$, traders will eventually make announcements that are within A^i_ω, the set of $E_p[X]$ for all beliefs p that have zero cost given ω. Because $\mathcal{P}_0$ is fixed for all $r \in T$, set A^i_ω is independent of the previous announcement. With Uncertainty Averse preferences, however, this is not the case. Let

$$A^i_{\omega,z} = \left\{ E_p[X] : G_{\Pi_i(\omega)}\left(r, p\right) = r, \text{ where } r = E_p\left(s(E_p[X], X) - s\big(z, X\big)\right) \right\}$$

be the set of all myopic announcements at ω given previous announcement z and beliefs p, that have zero cost. The set is nonempty for Variational and Smooth Ambiguity preferences because there is at least one belief p such that $G(r, p) = r$ for all $r \in T$.

We now argue that if $A^i_{\omega,z}$ depends on the previous announcement, information may not get aggregated. For simplicity, we consider the non-strategic environment but a similar argument applies to the strategic environment. Note first that, as in the proof of Theorem 3 with Uncertainty Averse preferences, traders will eventually make announcements in $A^i_z = \bigcap_{\omega' \in \mathcal{F}(\omega)} A^i_{\omega',z}$, as $t_k \to t_\infty$.

Suppose that $A^i_{z_{i-1}} \neq \emptyset$ for all $i \in I$ and $\bigcap_{j \in I} A^j_{z_{j-1}} \neq \emptyset$, where z_{j-1} is the previous announcement when j announces, which is Case 2 in the proof of Theorem 1 of the main paper. Suppose also that security X pays differently across at least two states in the common knowledge event $\mathcal{F}(\omega)$. With Variational preferences, because $A^j_{z_{j-1}}$ does not depend on z_{j-1}, we can fix $z \in \bigcap_{j \in I} A^j_{z_{j-1}} \neq \emptyset$ as the previous announcement for all traders. From the third point of Lemma 5, if the previous announcement is $z \in A^i_{z_{i-1}}$, then Trader i will repeat it. We then have that $d_c(\Pi_i(\omega'), z) = z$ for all $i \in I$ and $\omega' \in \bigcup_{p \in \mathcal{P}_{\mathcal{F}(\omega)}} Supp(p)$. But this implies that X is not strongly separable, a contradiction. With general Uncertainty Averse preferences, however, fixing $z \in \bigcap_{j \in I} A^j_{z_{j-1}}$ as the previous announcement for all traders may not have the same effect because it could be that $A^j_{z_{j-1}} \neq A^j_z$. If $z \notin A^j_z$, then Trader j will not announce z. This means that X may be strongly separable but traders never agree on the announcement, hence there is no information aggregation.

2 Existence of Revision-Proof Equilibrium

Revision-Proof equilibria do not always exist in games with an infinite horizon, due to dynamic inconsistency. This is true also in the case of complete information games with time inconsistency and infinite actions, as in Asheim (1997) and Ales and Sleet (2014). In this section, we show that if the game is continuous at infinity, then a Revision-Proof equilibrium exists. Continuity at infinity is achieved by shortening the time period, t_k, as $k \to \infty$, so that the discount factor decreases. This is similar to the approach of Ostrovsky (2012).

We first establish that a Consistent-Planning equilibrium always exists in games with finitely many actions and periods, using the results of Schlag and Zapechelnyuk (2020). We then assume that at least one Consistent-Planning equilibrium is strict, so that the one-shot best response at each information set is unique, given the fixed strategy of the future selves. We argue that this is a mild assumption. In the last round, everyone plays the myopic best response, which is unique from Lemma 1 of the main paper, hence the condition for strict equilibria is satisfied. In previous rounds, we have the freedom to determine off-equilibrium beliefs that will make deviations strictly suboptimal. We illustrate this using a specific example. We then show that a strict Consistent-Planning equilibrium is Revision-Proof. Finally, by adapting the proofs of Fudenberg and Levine (1983, 1986), we approximate the infinite game with a sequence of finite games and show that the sequence of Revision-Proof equilibria in the finite games converges to a Revision-Proof equilibrium in the infinite game.

2.1 Finite games

Recall that a game is a tuple $\Gamma^S(\Omega, I, \Pi, X, \mathcal{P}, y_0, \underline{y}, \overline{y}, s, \beta)$. An assessment $\mathcal{A} = \{\sigma, \mathscr{P}\}$ is a strategy profile $\sigma \in \Sigma$ and a system of beliefs $\mathscr{P} = \{\mathcal{P}(\mathcal{I})\}_{\mathcal{I} \in \mathscr{I}}$, where $\mathscr{I}$ is the collection of all information sets $\mathcal{I}$ and each $\mathcal{P}(\mathcal{I})$ is compact and convex. Assessment $\mathcal{A} = \{\sigma, \mathscr{P}\}$ is consistent if the system of beliefs is generated by the prior-by-prior updating of the set of common priors $\mathcal{P}$, given the strategy σ. As with Ostrovsky (2012), we discretise the action space, so that each trader can only choose among a finite set of possible announcements $\mathcal{Y} \subseteq [\underline{y}, \overline{y}]$. We denote a game with $k < \infty$ periods and finitely many actions as Γ^k. We denote a game with infinitely many periods and finitely many actions as Γ^∞.

We repeat below the definition of a Consistent-Planning equilibrium, which checks for one-shot deviations, as the trader considers the strategies of her future selves to be fixed. If the one-shot best response at each information set is unique, we say that the Consistent-Planning equilibrium is strict.

Definition 11. *Consistent pair $(\sigma^*, \mathscr{P})$ is a Consistent-Planning equilibrium if there is no information set $\mathcal{I}_k$, player $a_k = i$ and alternative strategy $\sigma = (\sigma_i, \sigma^*_{-i})$ with $\sigma_{i,k'} = \sigma^*_{i,k'}$ for all $k' \neq k$, such that*

$$V_i(\mathcal{I}_k, \sigma, \mathscr{P}) > V_i(\mathcal{I}_k, \sigma^*, \mathscr{P}).$$

It is a strict Consistent-Planning equilibrium if, for each information set $\mathcal{I}_k$, there is no alternative strategy $\sigma = (\sigma_i, \sigma^*_{-i})$ with $\sigma_{i,k'} = \sigma^*_{i,k'}$ for all $k' \neq k$, such that

$$V_i(\mathcal{I}_k, \sigma, \mathscr{P}) = V_i(\mathcal{I}_k, \sigma^*, \mathscr{P}).$$

We first establish that every finite game Γ^k has a Consistent-Planning equilibrium.

Proposition 7. *For every finite game Γ^k with $k < \infty$, there exists a Consistent-Planning equilibrium.*

This is a direct consequence of the proof of Theorem 1 in Schlag and Zapechelnyuk (2020). They have a very similar setting, consisting of a finite game with players who have sets of beliefs and update prior-by-prior. The only difference is that their utility function minimizes the maximum loss, whereas in our setting the players have MEU preferences. However, the only place in the proof where the utility function plays a role is to show that it is continuous with respect to the strategy of a player who only moves at an information set, and with respect to the strategies of everyone else who moves later, including her future selves. This is also true in our model. Excluding that difference, their equilibrium notion is essentially that of Consistent-Planning, so we have the result.

We now show that a strict Consistent-Planning equilibrium is Revision-Proof in finite games. We first restate the definition of a Revision-Proof equilibrium.

Definition 12. *Consistent pair $(\sigma^*, \mathscr{P})$ is a Revision-Proof equilibrium if there is no information set $\mathcal{I}_k$, player $a_k = i$ and alternative strategy $\sigma = (\sigma_i, \sigma^*_{-i})$, such that for all information sets $\mathcal{I}$ that are reachable from $\mathcal{I}_k$ and where i makes an announcement,*

$$V_i(\mathcal{I}, \sigma, \mathscr{P}) \geq V_i(\mathcal{I}, \sigma^*, \mathscr{P}),$$

with the inequality strict for at least one $\mathcal{I}$.

Note that this equilibrium notion is slightly stronger than the Revision-Proof equilibrium we have defined in the main paper, because here we check deviations at all information sets, not only those that are reached in the equilibrium path. All of the existence results in this section for Revision-Proof equilibria hold for this stronger version and therefore also for the the weaker version of the main paper.

Lemma 9. *If $(\sigma^*, \mathscr{P})$ is a strict Consistent-Planning equilibrium in game Γ^m, then it is a Revision-Proof equilibrium.*

Proof. Suppose not, so that there is information set $\mathcal{I}_k$, player $a_k = i$ and alternative strategy $\sigma = (\sigma_i, \sigma^*_{-i})$, such that for all information sets $\mathcal{I}$ that are reachable from $\mathcal{I}_k$ and where i makes an announcement,

$$V_i(\mathcal{I}, \sigma, \mathscr{P}) \geq V_i(\mathcal{I}, \sigma^*, \mathscr{P}),$$

with the inequality strict for at least one $\mathcal{I}$.

In the last period t_m, where i makes an announcement, we have $\sigma_{i,m} = \sigma^*_{i,m}$ from the definition of a strict Consistent-Planning equilibrium. In period t_{m-n}, where i makes her penultimate announcement and given that at t_m we have $\sigma_{i,m} = \sigma^*_{i,m}$, the strictness of Consistent-Planning equilibrium implies that i has a unique best response at t_{m-n} and $\sigma_{i,m-n} = \sigma^*_{i,m-n}$. This means that, given that the future self at t_m has a unique optimal action at each information set given her beliefs, it is not possible for i at t_{m-n} to suggest to her future self to deviate to another action that she is indifferent to. Given this restriction, the strictness of the Consistent-Planning equilibrium implies that at t_{m-n} her optimal action is also unique. Going backwards until $\mathcal{I}_k$, there is no alternative strategy σ that will make all future selves of i weakly better off and at least one strictly better off, which is a contradiction. $\square$

We now make the assumption that among the set of all Consistent-Planning equilibria, at least one is strict.

Assumption 3. *For every finite game Γ^k with $k < \infty$, if the set of Consistent-Planning equilibria is nonempty, then at least one is strict.*

We argue that this is a mild assumption. First, everyone plays the myopic best response in the last round, which is unique from Lemma 1 of the main paper, hence the condition for strict equilibria is satisfied. Second, in previous rounds, we have the freedom to determine off-equilibrium beliefs that will make deviations strictly suboptimal.

We now show an example where everyone playing the unique myopic best response is a strict Consistent planning and therefore Revision-Proof equilibrium, that aggregates information for any discount factor. Note that, in standard finite games with Expected Utility, two states with conditionally independent signals, and a logarithmic market scoring rule, Chen et al. (2010) show that there is a unique weak Perfect Bayesian equilibrium, where everyone plays their unique myopic best response in every round.

Suppose there are four states and security X pays $\{1, 2, 3, 4\}$, so we refer to states in terms of their payoffs. Ann's partition is $\{\{1, 2\}, \{3, 4\}\}$, whereas Bob's is $\{\{1, 4\}, \{2, 3\}\}$.

State	Ann	Bob	Ann
1	1	1	1
2	1	2	2
3	3	3	3
4	3	4	4

Table 1: Announcements in Revision-Proof Equilibrium

They have a common set of priors, consisting of all probability measures, $\Delta(\Omega)$.[7]There are three periods, so Ann announces first, then Bob, and then Ann. The initial announcement by the market maker is 1.

We show that everyone playing their myopic best response is a strict Consistent-Planning and therefore Revision-Proof equilibrium. At $\{1, 2\}$, Ann announces 1, whereas at $\{3, 4\}$, she announces 3, thus revealing her type to Bob.[8] At state 1, Ann announces 1 and reveals to Bob that the state is not 4. Hence, Bob knows that the state is 1 and announces 1, revealing to Ann the true state, so she repeats the announcement. Table 2.1 specifies the myopic announcements at each state.

Suppose that at state 1, Ann deviates by mixing between the two myopic announcements, 1 and 3. A deviation of 3 is interpreted by Bob as Ann being a $\{3, 4\}$ type, hence he announces 4. In the last period, Ann is at an off-equilibrium information set, and we assume that she also believes that she is a $\{3, 4\}$ type, whereas Bob is a $\{1, 4\}$ type, hence she repeats 4. This means that Ann in the last period gets a zero payoff. In the first period, her deviation of 3 provides a lower payoff than the myopic best of 1. Hence, she does not want to deviate. Any announcement z, different from 1 and 3, leads to an off-equilibrium information set for both Bob and Ann in the last two periods. We then assume that they form a unique belief p so that $E_p[X] = z$, hence they repeat z in periods 2 and 3. As a result, Ann in the first period gets a lower payoff than the myopic best but in the last period she gets zero. Hence, she has no incentive to deviate. In periods 2 and 3, Ann and Bob announce for the last time so they play their myopic best response. Similar arguments apply to all other states, so this is a Revision-Proof equilibrium.

The myopic is also a Revision-Proof equilibrium if we have more than three periods. From period 3, at all states the true value of the security is revealed and Ann agrees with Bob on the announcement. The only difference from the three-period game is that Bob in period 2 is strategic, as he announces at least once more. Note that Bob learns and announces the true

[7]We assume the set of all priors $\Delta(\Omega)$ for simplicity, so that the myopic best response is always one of the values of X, $\{1, 2, 3, 4\}$. The only drawback of the example is that some beliefs assign probability zero to the true state.

[8]Recall that, from Lemma 1 in the main paper, Ann's myopic best response is to announce as close as possible to the previous announcement, given her posterior beliefs.

value of X in period 2, at all states. If at state 1 he deviated by announcing $z \in \{2, 3, 4\}$, Ann would repeat z, thinking this is the true value. Bob in periods 4 and above is off-equilibrium and repeats the previous announcement, using the same beliefs we constructed for Ann in the three-period game. Hence, Bob in period 2 is worse off by not announcing the myopically best, whereas he does not gain anything from period 4 onwards.

Combining Proposition 7, Assumption 3, and Lemma 9, we have the following lemma.

Lemma 10. *For every finite game Γ^k with $k < \infty$, there exists a Revision-Proof equilibrium.*

2.2 Truncated games with finitely many periods

For each game Γ^∞ with infinitely many periods, we will generate a sequence of truncated games with finitely many periods. Following Fudenberg and Levine (1983, 1986), we assume, without loss of generality, that there is a "do nothing" or null action at each time t, denoted 0. The 0 action means that the trader repeats, with probability 1, the previous announcement. Note that the null action guarantees that the agent's payoff at that period is 0, irrespective of the previous announcement. With each game Γ^∞, we associate a collection of truncated games. A truncated game $\Gamma(m)$ effectively ends in time t_m, because for all $t_k > t_m$, the only available action for any trader is the null action of repeating the previous announcement and their beliefs no longer update. Hence, the announcement in t_m is repeated by everyone. Note that a game Γ^m ends at t_m, whereas game $\Gamma(m)$ has infinitely many periods but traders can only choose the null action after t_m.

More formally, let $\Sigma(m)$ be the strategy space of truncated game $\Gamma(m)$. Each strategy profile $\sigma \in \Sigma(m)$ is of the form $(\sigma_1, \sigma_2, \ldots, \sigma_m, 0, 0, \ldots)$, where σ_k maps information sets at time t_k to the set of possible (finite) announcements $\mathcal{Y}$, for the agent who makes an announcement at t_k. Note that $\Sigma(1) \subseteq \Sigma(2) \subseteq \ldots \subseteq \Sigma(\infty)$, where $\Sigma(\infty) = \Sigma$ is the strategy space of the infinite horizon game. Moreover, each $\Sigma(m)$ is compact.

Let $\mathcal{A}(m)$ be the collection of assessments $\mathcal{A} = (h, \mathscr{P})$ such that $h \in \Sigma(m)$ and beliefs do not update after m, so that if $k > m$ and information set $\mathcal{I}_{k-1}$ immediately precedes $\mathcal{I}_k$, then $\mathcal{P}(\mathcal{I}_k) = \mathcal{P}(\mathcal{I}_{k-1})$. Let $\mathscr{P}(m)$ be the collection of all systems of beliefs $\mathscr{P}$ of game $\Gamma(m)$ and note that $\mathscr{P}(m)$ is compact.

An assessment $\mathcal{A} = (h, \mathscr{P})$ is an equilibrium in $\Gamma(m)$ only if $\mathcal{A} = (h, \mathscr{P}) \in \mathcal{A}(m)$. Note that $\mathcal{A}(1) \subseteq \mathcal{A}(2) \subseteq \ldots \subseteq \mathcal{A}(\infty)$, where $\mathcal{A}(\infty)$ is the collection of all assessments in the infinite horizon game Γ^∞. Intuitively, after t_m no trader changes her action, hence there is no information revelation and beliefs do not update. Set $\mathcal{A}(\infty)$ is compact because it is a closed subset of $\times_{m=1}^{\infty}(\Sigma(m) \times \mathscr{P}(m))$, which is the product of compact sets in the product topology, and therefore also compact.

2.3 Continuity

In order to approximate an equilibrium in the infinite game Γ^∞ we need to define the distance between assessments. Recall that Φ is the space of all uncertainty and let $\delta(p,q) = \sup_{E\in\Phi}|p(E)-q(E)|$ be the distance between two beliefs over Φ. The Hausdorff distance between two sets of measures $\mathcal{P}$, $\mathcal{P}'$ is

$$\delta(\mathcal{P},\mathcal{P}') = \max\left\{\sup_{p\in\mathcal{P}}\delta(p,\mathcal{P}'), \sup_{p'\in\mathcal{P}'}\delta(\mathcal{P},p')\right\},$$

where $\delta(q,\mathcal{P}) = \inf_{p\in\mathcal{P}}\delta(q,p)$. The set of all nonempty compact subsets of Φ endowed with the Hausdorff topology is a compact metric space (Theorem 3.71(3) in Aliprantis and Border (2013)).

A system of beliefs specifies a compact set of beliefs for each information set. We define the distance between two systems of beliefs $\mathscr{P}, \mathscr{P}'$ as

$$d(\mathscr{P},\mathscr{P}') = \sup_{\mathcal{I}_k\in\mathscr{I}}\left\{\frac{1}{k}\min\{\delta(\mathcal{P}(\mathcal{I}_k),\mathcal{P}'(\mathcal{I}_k)),1\}\right\}.$$

This metric is motivated in Fudenberg and Levine (1983). It specifies that two systems of beliefs are close to each other if they only differ in the distant future. We can similarly measure the distance between two strategies σ,σ' with $d(\sigma,\sigma') = \sup_{\mathcal{I}_k\in\mathscr{I}}\left\{\frac{1}{k}\min\{\delta(\sigma(\mathcal{I}_k),\sigma'(\mathcal{I}_k)),1\}\right\}$, where $\sigma(\mathcal{I}_k)$ is the strategy of the trader who announces at t_k and information set $\mathcal{I}_k$. Finally, we can also measure the distance between truncated systems of beliefs (and strategies) that occur after an information set. Given an information set $\mathcal{I}_k$, let $\mathcal{I}_k(\mathscr{P})$ be the restriction of $\mathscr{P}$ to all information sets that succeed $\mathcal{I}_k$, including $\mathcal{I}_k$. The same notation $\mathcal{I}_k(\sigma)$ applies to a strategy σ. Then, the distances $d(\mathcal{I}_k(\mathscr{P}),\mathcal{I}_k(\mathscr{P}'))$ and $d(\mathcal{I}_k(\sigma),\mathcal{I}_k(\sigma'))$ calculate the distance only with respect to information sets that follow $\mathcal{I}_k$.

We can now define the distance between two assessments $\mathcal{A} = \{\sigma,\mathscr{P}\}, \mathcal{A}' = \{\sigma',\mathscr{P}'\}$:

$$d(\mathcal{A},\mathcal{A}') \equiv \sup_{\mathcal{I}_k\in\mathscr{I}}\left\{d(\mathcal{I}_k(\sigma),\mathcal{I}_k(\sigma')), \left\{\sup_{h_{a_k}\in\Sigma_{a_k}} d(\mathcal{I}_k(h_{a_k},\sigma_{-a_k}),\mathcal{I}_k(h_{a_k},\sigma'_{-a_k}))\right\}, d(\mathcal{I}_k(\mathscr{P}),\mathcal{I}_k(\mathscr{P}'))\right\},$$

where a_k is the announcer at time t_k and information set $\mathcal{I}_k$.

This metric is also motivated in Fudenberg and Levine (1983). Two assessments $\mathcal{A}$ and $\mathcal{A}'$ are close to each other if the following three conditions are true at each information set $\mathcal{I}_k$. First, the distributions over actions that are generated by σ and σ' for every subsequent information set are close to each other. Second, the distributions over actions are close to each other even when trader a_k deviates with h_{a_k} at $\mathcal{I}_k$. Finally, the sequence of sets of beliefs that are generated from $\mathscr{P}$, $\mathscr{P}'$ given $\mathcal{I}_k$ are also close to each other.

Given an assessment $\mathcal{A} = \{\sigma, \mathscr{P}\}$, let $V_i(\mathcal{I}, \sigma, \mathscr{P})$ be i's continuation payoff at information set $\mathcal{I}$. We say that the game is uniformly continuous if whenever two assessments are close to each other, the continuation payoffs are also close, for each information set.

Definition 13. *The game Γ^∞ is uniformly continuous if for all information sets $\mathcal{I}$ and all sequences of assessments $\{\mathcal{A}^n\} = \{\sigma^n, \mathscr{P}^n\}$, $\{\mathcal{A}'^n\} = \{\sigma'^n, \mathscr{P}'^n\}$, $\mathcal{A}^n \to \mathcal{A}'^n$ implies $\left|V_i(\mathcal{I}, \sigma^n, \mathscr{P}^n) - V_i(\mathcal{I}, \sigma'^n, \mathscr{P}'^n)\right| \to 0$ for all $i \in I$.*

Ostrovsky (2012) generates a uniformly continuous game by shortening the time period t_m as $m \to \infty$. For example, in his version of the Kyle (1985) model, he sets $t_k = 1 - \frac{1}{2^k}$. Although he does not specify how he achieves uniform continuity in the prediction market model, we use something similar. Following Dimitrov and Sami (2008), we assume that the payment at t_m is $\beta^m(s(y_{t_m}, x^*) - s(y_{t_{m-1}}, x^*))$, instead of $s(y_{t_m}, x^*) - s(y_{t_{m-1}}, x^*)$. This is equivalent to shortening the period t_m as $m \to \infty$, so that the discounting factor needs to decrease accordingly. Although the results in the main paper are true with both definitions, the former ensures that the game is uniformly continuous. We refer to this uniformly continuous game with finitely many announcements as Γ^∞.

Let $\mathcal{I}_{k\leq m}$ be the collection of all information sets $\mathcal{I}_k$ at times $t_k \leq t_m$ and $\mathscr{P}(\infty)$ the collection of all systems of beliefs. Let constant w^m be the greatest variation in any agent's payoff and for any system of beliefs, from strategies σ, τ that are identical for all information sets up to time t_{m-1}, written as $\sigma =_{m-1} \tau$.

$$w^m \equiv \sup_{\substack{\mathcal{I}\in\mathscr{I} \\ \mathscr{P}\in\mathscr{P}(\infty)}} \sup_{\substack{i\in I \\ \sigma =_{m-1}\tau}} \left|V_i(\mathcal{I}, \sigma, \mathscr{P}) - V_i(\mathcal{I}, \tau, \mathscr{P})\right|.$$

Definition 14. *The game Γ^∞ is continuous at infinity if $w^m \to 0$ as $m \to \infty$.*

It is straightforward that because Γ^∞ is uniformly continuous, it is also continuous at infinity.

2.4 Existence of equilibrium in Γ^∞

Following Fudenberg and Levine (1983, 1986), we prove existence in the infinite game Γ^∞ through a series of lemmas. We first define the notion of an ϵ-Revision-Proof equilibrium.

Definition 15. *Consistent pair $(\sigma^*, \mathscr{P})$ is an ϵ-Revision-Proof equilibrium if there is no information set $\mathcal{I}_k$, player $a_k = i$ and alternative strategy $\sigma = (\sigma_i, \sigma^*_{-i})$, such that for all* *information sets $\mathcal{I}$ that are reachable from $\mathcal{I}_k$ and where i makes an announcement,*

$$V_i(\mathcal{I}, \sigma, \mathscr{P}) \geq V_i(\mathcal{I}, \sigma^*, \mathscr{P}) + \epsilon,$$

with the inequality strict for at least one $\mathcal{I}$.

Lemma 11. *If $(h^*, \mathscr{P}) \in \mathcal{A}(m)$ is an ϵ-Revision-Proof equilibrium in $\Gamma(m)$, then $(h^*, \mathscr{P})$ is an $(\epsilon + w^m)$-Revision-Proof equilibrium in Γ^∞.*

Proof. Suppose $(h^*, \mathscr{P})$ is an ϵ-Revision-Proof in $\Gamma(m)$ and let $g \in \Sigma(\infty)$. Set $h = (g_1, g_2, \ldots, g_m, 0, \ldots)$. There are two cases. First, for any information set $\mathcal{I} \in \mathscr{I}$ where agent i makes an announcement, we have

$$V_i(\mathcal{I}, h_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}, h^*, \mathscr{P}) \leq \epsilon.$$

Because h and g differ only after t_m, we have

$$V_i(\mathcal{I}, g_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}, h_i, h^*_{-i}, \mathscr{P}) \leq w^m.$$

Adding the two inequalities we have

$$V_i(\mathcal{I}, g_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}, h^*, \mathscr{P}) \leq \epsilon + w^m.$$

Second, there is information set $\mathcal{I}$ and information set $\mathcal{I}'$ that is reachable from $\mathcal{I}$, both before time t_m, where i makes the announcement and we have $V_i(\mathcal{I}, h_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}, h^*, \mathscr{P}) > \epsilon$ but $V_i(\mathcal{I}', h_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}', h^*, \mathscr{P}) < \epsilon$, so that the future self at $\mathcal{I}'$ will not follow i's recommendation at $\mathcal{I}$ when considering an ϵ-Revision-Proof equilibrium. Because h and g differ only after t_m, we have $V_i(\mathcal{I}', g_i, h^*_{-i}, \mathscr{P}) - V_i(\mathcal{I}', h^*, \mathscr{P}) < \epsilon + w^m$, hence this deviation would not be followed by a future self at $\mathcal{I}'$ when considering an $(\epsilon + w^m)$-Revision-Proof equilibrium.

For any information set $\mathcal{I}_k$ where $t_k \leq t_m$, the conditions on beliefs are satisfied because $(h^*, \mathscr{P})$ is an ϵ-Revision-Proof equilibrium in $\Gamma(m)$. For any other information set, the conditions on beliefs are also satisfied because everyone chooses the null action of repeating the previous announcement, hence there is no updating of information or beliefs. □

Lemma 12. *Consider a sequence of assessments $\{\mathcal{A}_m\} = \{g_m, \mathscr{P}_m\}$ such that each $\mathcal{A}_m$ is an ϵ-Revision-Proof equilibrium in Γ^∞ and $\mathcal{A}_m \to \mathcal{A} = \{g, \mathscr{P}\}$. Then, $\mathcal{A}$ is an ϵ-Revision-Proof equilibrium in Γ^∞.*

Proof. First, note that since $\mathcal{A}_m \to \mathcal{A} = (g, \mathscr{P})$, the corresponding sets of beliefs $\mathscr{P}_m$ converge to $\mathscr{P}$, hence the consistency condition on beliefs is satisfied. Suppose there is an information set $\mathcal{I}$ and a strategy h_i such that

$$V_i(\mathcal{I}, h_i, g_{-i}, \mathscr{P}) - V_i(\mathcal{I}, g, \mathscr{P}) \geq \epsilon + 3\delta,$$

and for all $\mathcal{I}'$ that are reachable from $\mathcal{I}$ and i makes an announcement, we have

$$V_i(\mathcal{I}', h_i, g_{-i}, \mathscr{P}) - V_i(\mathcal{I}', g, \mathscr{P}) \geq \epsilon.$$

Because $\mathcal{A}_m \to \mathcal{A}$ and the game is uniformly continuous, for any δ, there exists large m such that

$$V_i(\mathcal{I}, g_m, \mathscr{P}_m) - V_i(\mathcal{I}, g, \mathscr{P}) < \delta.$$

$$V_i(\mathcal{I}, h_i, g_{-i}, \mathscr{P}) - V_i(\mathcal{I}, h_i, g_{m-i}, \mathscr{P}_m) < \delta,$$

and the same inequalities hold for all $\mathcal{I}'$ that are reachable from $\mathcal{I}$ and i makes an announcement.

Combining the three inequalities for $\mathcal{I}$ and for $\mathcal{I}'$, we have

$$V_i(\mathcal{I}, h_i, g_{m-i}, \mathscr{P}_m) - V_i(\mathcal{I}, g_m, \mathscr{P}_m) > \epsilon + \delta,$$

$$V_i(\mathcal{I}', h_i, g_{m-i}, \mathscr{P}_m) - V_i(\mathcal{I}', g_m, \mathscr{P}_m) > \epsilon - 2\delta.$$

As δ can be taken to be arbitrarilly small, we can find big enough m so that

$$V_i(\mathcal{I}, h_i, g_{m-i}, \mathscr{P}_m) - V_i(\mathcal{I}, g_m, \mathscr{P}_m) > \epsilon,$$

$$V_i(\mathcal{I}', h_i, g_{m-i}, \mathscr{P}_m) - V_i(\mathcal{I}', g_m, \mathscr{P}_m) \geq \epsilon,$$

which contradicts that $\mathcal{A}_m$ is an ϵ-Revision-Proof equilibrium. □

Lemma 13. *Suppose that there is a sequence $\{\mathcal{A}_m\} = \{g_m, \mathscr{P}_m\}$ such that $\mathcal{A}_m$ is an ϵ_m-Revision-Proof equilibrium in $\Gamma(m)$ and, as $m \to \infty$, we have $\epsilon_m \to 0$ and $\mathcal{A}_m \to \mathcal{A}^* = (g^*, \mathscr{P}^*)$. Then, $\mathcal{A}^*$ is a Revision-Proof equilibrium in Γ^∞.*

Proof. From Lemma 11, $\mathcal{A}_m$ is an $(\epsilon_m + w^m)$-Revision-Proof equilibrium in the infinite game Γ^∞. Because Γ^∞ is uniformly continuous, it is also continuous at infinity and $\epsilon_m + w^m \to 0$. We therefore have that for each $\delta > 0$ there is M such that $\epsilon_m + w^m < \delta$, whenever $m > M$. From Lemma 12, $\mathcal{A}^*$ is a δ-Revision-Proof equilibrium in the infinite game Γ^∞. Since this is true for every $\delta > 0$, $\mathcal{A}^*$ is a Revision-Proof equilibrium in Γ^∞. □

Proposition 8. *There is a Revision-Proof equilibrium in Γ^∞.*

Proof. From Lemma 10, each finite-horizon Γ^m and therefore $\Gamma(m)$ has a Revision-Proof equilibrium $\mathcal{A}_m$, for any $m < \infty$. From Lemma 11, $\mathcal{A}_m$ is a w^m-Revision-Proof equilibrium in Γ^∞. Since $\mathcal{A}(\infty)$ is compact, there is a subsequence $\mathcal{A}_k \subseteq \mathcal{A}_m$ with $\mathcal{A}_k \to \mathcal{A}^*$. From Lemma 13, $\mathcal{A}^*$ is a Revision-Proof equilibrium in Γ^∞. $\square$

3 Experimental Instructions

The purpose of this experimental session is to study how people make decisions in a particular situation. Your earnings will depend upon the decisions you make as well as the decisions that other people make. At the end of the session, you will be paid in cash your total earnings. None of the other participants will be informed of your earnings, and likewise you will not be informed of the earnings of others. Given that nobody will know of each other's identity, all the decisions you make during the experimental session will be anonymous.

For your participation in the experimental session, you will receive an initial payment of 6,000 Experimental Currency Units (ECUs), which will be converted into euros at an exchange rate of 2,000 ECUs equal €1.

The experimental session consists of **3 parts** to be described at the appropriate time.

The instructions are simple. If you have a question, please raise your hand. Aside from these questions, any communication with other participants or looking at other participants' screens is not permitted and will lead to your immediate exclusion from the experimental session.

<u>The instructions are identical to all participants.</u>

3.1 Part 1

In this part of the study, you are asked to choose one of the five options shown below. Regardless of which option you choose, there are two possible outcomes (Outcome A and Outcome B). These outcomes are equally likely for all five options; that is, there is a 50% chance of Outcome A and a 50% chance of Outcome B, just like the flip of a coin. The options differ only in how much each outcome pays. The table below tells you how much you will be paid for each outcome. The computer will randomly choose between Outcome A and Outcome B at the end of the experimental session. You can imagine the computer flipping a virtual coin so that the chance of each outcome is equal. You will only find out your outcome from Part 1 and how much you will be paid for Part 1 at the end of the experimental session. Please choose your option by clicking on a radio button.

Option	Outcome	Payoff	Probabilities
1	A	2,000 ECUs	50%
	B	2,000 ECUs	50%
2	A	1,400 ECUs	50%
	B	3,500 ECUs	50%
3	A	1,000 ECUs	50%
	B	4,500 ECUs	50%
4	A	600 ECUs	50%
	B	5,500 ECUs	50%
5	A	200 ECUs	50%
	B	6,500 ECUs	50%

3.2 Part 2

3.2.1 Red = 30, Green = 30, Blue = 30, Red → High [EUS0]

Recall that the instructions are identical to all participants.

You are about to participate in an experiment about prediction markets. You will spend the next few minutes learning how to make predictions and how your earnings are calculated. All values are denominated in Experimental Currency Units (ECUs). With the completion of this part, your ECUs will be converted into euros at the exchange rate of 2,000 ECUs equals €1.

In each round, you have at your disposal 1,500 ECUs; that is, in each round you can **only** use your starting 1,500 ECUs. There are 12 rounds of game play. Though for the duration of the round you will be paired with the **same** participant, in every new round, you will be matched with a **different** participant. Within a given round, the pair of participants will take on the role of traders that take turns (alternate at) predicting the value of the stock. Specifically, **within a given round**, first, Trader 1 will provide his prediction for the value of the stock, then Trader 2 will provide her prediction for the value of the stock, then Trader 1 will provide his prediction for the value of the stock, then Trader 2, and so on and so forth. Whether or not you are Trader 1 or Trader 2 will be determined by a computer draw **at the beginning of each round**. You can imagine the computer flipping a virtual coin so that the chance of each outcome is equal.

How many predictions within a given round will the two traders report? Again, this is determined by a computer draw. Specifically, after the report of each prediction, the computer will draw an integer from 1 to 100 (all inclusive), where each integer has the same probability of being drawn. If the computer draws an integer below or equal to 95, then there will be one more prediction in the round; otherwise, if the computer draws an integer above 95, then the round ends. Thus, after each prediction, there is 95% chance that there will be one more prediction in the round, and 5% chance that there will be no other prediction and the round will end.

Your prediction for the value of the stock can be any integer from 0 to 100.

To help you decide on your prediction, we will provide next the payoff functions and some information about the value of the stock.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

Let's look at these payoff functions more closely.

When the value of the stock is high, assuming you just reported, then *your prediction* must **exceed** that of the *previous trader's reported prediction* to make **profits**. Why? $(100 - \textit{your prediction})^2$ is smaller than $(100 - \textit{previous trader's reported prediction})^2$ precisely because *your prediction* is a bigger number than the *previous trader's reported prediction.* Therefore, $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2] > 0$. Otherwise, when the value of the stock is high, and *your prediction* is **less than** that of the *previous trader's reported prediction*, you will make **losses**.

When the value of the stock is low, the opposite is true. Assuming you just reported, then *your prediction* must **be less than** that of the *previous trader's reported prediction* to make **profits**. Otherwise, when the value of the stock is low, and *your prediction* **exceeds** that of the *previous trader's reported prediction*, you will make **losses**.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock**

value is revealed to you. Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

Examples

For all the calculations in the examples, assume the following.

Payoff Functions
The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction*.

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock value is revealed to you.** Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

1. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 50.00, and yours is 60.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −11.00 ECUs

2. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 55.00, and yours is 35.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 18.00 ECUs

3. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 67.00, and yours is 72.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −6.95 ECUs

4. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 77.00, and yours is 22.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 54.45 ECUs

Quiz

For the calculations, use the calculator provided in the bottom left portion of this screen. Press the icon and the calculator will become live. To use the scientific calculator, press view and choose the scientific calculator. Provide your numerical answers to two decimal places.

Recall that:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - previous\ trader's\ reported\ prediction)^2 - (100 - your\ prediction)^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(previous\ trader's\ reported\ prediction)^2 - (your\ prediction)^2]$.

A colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

We provide the two traders with some **private** information about the draw. This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

1. How many rounds of game play are there? 12.00

2. During the duration of the round, you will be matched with the same individual. Yes

3. In each round, you will be matched with the same individual. No

4. To determine whether there will be another prediction in the round, the computer draws integer 27. Is there going to be another prediction in the round? Yes

5. To determine whether there will be another prediction in the round, the computer draws integer 96. Is there going to be another prediction in the round? No

6. The round payoff is determined at the very end of the round, when the value of the stock is revealed to you. Yes

7. If your round payoff turns out to be negative, then we will zero your round payoff for that round. Yes

8. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 57.00, and yours is 82.00. What is your payoff for this prediction? -34.75 ECUs

9. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 57.00, and yours is 85.00. What is your payoff for this prediction? -39.76 ECUs

10. In the beginning of the round, you, as Trader 1, received private information that the ball is not blue. Suppose that Trader 2's previous prediction was 40.00, and yours is also 40.00. What is your payoff for this prediction? 0.00 ECUs

11. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 40.00, and yours is 50.00. What is your payoff for this prediction? -9.00 ECUs

12. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that your very first prediction is 30.00. What is your payoff for this prediction? -9.00 ECUs [The initial value of 0 was changed to 50 in the other set of treatments, which yields 16.00 ECUs.]

13. Suppose that the color of the ball is revealed to you at the end of the round, and you earned the following payoffs for your predictions within the round: 15.00, -10.00, 25.00, 40.00, 10.00. Recall that in the beginning of each round you are provided with 1,500 ECUs. What is your round payoff? 1,580.00 ECUs

14. Suppose that in the 12 rounds, you earned the following round payoffs: 1,500, 1,000, 2,000, 1,500, 1,000, 1,000, 1,000, 1,000, 1,000, 1,000, 2,000, 2,000. What is your final payoff? 16,000.00 ECUs

15. If you earned 16,000 ECUs in the 12 rounds, your final payoff in euros is what? €8.00

3.2.2 $0 \leq$ Red $\leq$ 30, $20 \leq$ Green $\leq$ 70, $20 \leq$ Blue $\leq$ 70, Red $\rightarrow$ High [AmbS0]

Recall that the instructions are identical to all participants.

You are about to participate in an experiment about prediction markets. You will spend the next few minutes learning how to make predictions and how your earnings are calculated. All values are denominated in Experimental Currency Units (ECUs). With the completion of this part, your ECUs will be converted into euros at the exchange rate of 2,000 ECUs equals €1.

In each round, you have at your disposal 1,500 ECUs; that is, in each round you can **only** use your starting 1,500 ECUs. There are 12 rounds of game play. Though for the duration of the round you will be paired with the **same** participant, in every new round, you will be matched with a **different** participant. Within a given round, the pair of participants will take on the role of traders that take turns (alternate at) predicting the value of the stock. Specifically, **within a given round**, first, Trader 1 will provide his prediction for the value of the stock, then Trader 2 will provide her prediction for the value of the stock, then Trader 1 will provide his prediction for the value of the stock, then Trader 2, and so on and so forth. Whether or not you are Trader 1 or Trader 2 will be determined by a computer draw **at the beginning of each round**. You can imagine the computer flipping a virtual coin so that the chance of each outcome is equal.

How many predictions within a given round will the two traders report? Again, this is determined by a computer draw. Specifically, after the report of each prediction, the computer will draw an integer from 1 to 100 (all inclusive), where each integer has the same probability of being drawn. If the computer draws an integer below or equal to 95, then there will be one more prediction in the round; otherwise, if the computer draws an integer above 95, then the round ends. Thus, after each prediction, there is 95% chance that there will be one more prediction in the round, and 5% chance that there will be no other prediction and the round will end.

Your prediction for the value of the stock can be any integer from 0 to 100.

To help you decide on your prediction, we will provide next the payoff functions and some information about the value of the stock.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \mathit{previous\ trader's\ reported\ prediction})^2 - (100 - \mathit{your\ prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\mathit{previous\ trader's\ reported\ prediction})^2 - (\mathit{your\ prediction})^2]$.

Let's look at these payoff functions more closely.

When the value of the stock is high, assuming you just reported, then *your prediction* must **exceed** that of the *previous trader's reported prediction* to make **profits**. Why? $(100 - \mathit{your\ prediction})^2$ is smaller than $(100 - \mathit{previous\ trader's\ reported\ prediction})^2$ precisely because *your prediction* is a bigger number than the *previous trader's reported prediction.* Therefore, $0.01[(100 - \mathit{previous\ trader's\ reported\ prediction})^2 - (100 - \mathit{your\ prediction})^2] > 0$. Otherwise, when the value of the stock is high, and *your prediction* is **less than** that of the *previous trader's reported prediction*, you will make **losses**.

When the value of the stock is low, the opposite is true. Assuming you just reported, then *your prediction* must **be less than** that of the *previous trader's reported prediction* to make **profits**. Otherwise, when the value of the stock is low, and *your prediction* **exceeds** that of the *previous trader's reported prediction*, you will make **losses**.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock**

value is revealed to you. Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: between 0 and 30 red balls, between 20 and 70 green balls, and between 20 and 70 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

Examples

For all the calculations in the examples, assume the following.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock value is revealed to you.** Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: between 0 and 30 red balls, between 20 and 70 green balls, and between 20 and 70 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

1. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 50.00, and yours is 60.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −11.00 ECUs

2. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 55.00, and yours is 35.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 18.00 ECUs

3. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 67.00, and yours is 72.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −6.95 ECUs

4. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 77.00, and yours is 22.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 54.45 ECUs

Quiz

For the calculations, use the calculator provided in the bottom left portion of this screen. Press the icon and the calculator will become live. To use the scientific calculator, press view and choose the scientific calculator. Provide your numerical answers to two decimal places.

Recall that:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \mathit{previous\ trader's\ reported\ prediction})^2 - (100 - \mathit{your\ prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\mathit{previous\ trader's\ reported\ prediction})^2 - (\mathit{your\ prediction})^2]$.

A colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. The virtual urn contains 90 balls: between 0 and 30 red balls, between 20 and 70 green balls, and between 20 and 70 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is low.
- If the drawn ball is blue, then the stock value is again low.

We provide the two traders with some **private** information about the draw. This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

1. How many rounds of game play are there? 12.00

2. During the duration of the round, you will be matched with the same individual. Yes

3. In each round, you will be matched with the same individual. No

4. To determine whether there will be another prediction in the round, the computer draws integer 27. Is there going to be another prediction in the round? Yes

5. To determine whether there will be another prediction in the round, the computer draws integer 96. Is there going to be another prediction in the round? No

6. The round payoff is determined at the very end of the round, when the value of the stock is revealed to you. Yes

7. If your round payoff turns out to be negative, then we will zero your round payoff for that round. Yes

8. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 57.00, and yours is 82.00. What is your payoff for this prediction? −34.75 ECUs

9. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 57.00, and yours is 85.00. What is your payoff for this prediction? −39.76 ECUs

10. In the beginning of the round, you, as Trader 1, received private information that the ball is not blue. Suppose that Trader 2's previous prediction was 40.00, and yours is also 40.00. What is your payoff for this prediction? 0.00 ECUs

11. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 40.00, and yours is 50.00. What is your payoff for this prediction? −9.00 ECUs

12. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that your very first prediction is 30.00. What is your payoff for this prediction? −9.00 ECUs [The initial value of 0 was changed to 50 in the other set of treatments, which yields 16.00 ECUs.]

13. Suppose that the color of the ball is revealed to you at the end of the round, and you earned the following payoffs for your predictions within the round: 15.00, −10.00, 25.00, 40.00, 10.00. Recall that in the beginning of each round you are provided with 1,500 ECUs. What is your round payoff? 1,580.00 ECUs

14. Suppose that in the 12 rounds, you earned the following round payoffs: 1,500, 1,000, 2,000, 1,500, 1,000, 1,000, 1,000, 1,000, 1,000, 1,000, 2,000, 2,000. What is your final payoff? 16,000.00 ECUs

15. If you earned 16,000 ECUs in the 12 rounds, your final payoff in euros is what? €8.00

3.2.3 Red = 30, Green = 30, Blue = 30, Red & Green $\rightarrow$ High [EUStS0]

Recall that the instructions are identical to all participants.

You are about to participate in an experiment about prediction markets. You will spend the next few minutes learning how to make predictions and how your earnings are calculated. All values are denominated in Experimental Currency Units (ECUs). With the completion of this part, your ECUs will be converted into euros at the exchange rate of 2,000 ECUs equals €1.

In each round, you have at your disposal 1,500 ECUs; that is, in each round you can **only** use your starting 1,500 ECUs. There are 12 rounds of game play. Though for the duration of the round you will be paired with the **same** participant, in every new round, you will be matched with a **different** participant. Within a given round, the pair of participants will take on the role of traders that take turns (alternate at) predicting the value of the stock. Specifically, **within a given round**, first, Trader 1 will provide his prediction for the value of the stock, then Trader 2 will provide her prediction for the value of the stock, then Trader 1 will provide his prediction for the value of the stock, then Trader 2, and so on and so forth. Whether or not you are Trader 1 or Trader 2 will be determined by a computer draw **at the beginning of each round**. You can imagine the computer flipping a virtual coin so that the chance of each outcome is equal.

How many predictions within a given round will the two traders report? Again, this is determined by a computer draw. Specifically, after the report of each prediction, the computer will draw an integer from 1 to 100 (all inclusive), where each integer has the same probability of being drawn. If the computer draws an integer below or equal to 95, then there will be one more prediction in the round; otherwise, if the computer draws an integer above 95, then the round ends. Thus, after each prediction, there is 95% chance that there will be one more prediction in the round, and 5% chance that there will be no other prediction and the round will end.

Your prediction for the value of the stock can be any integer from 0 to 100.

To help you decide on your prediction, we will provide next the payoff functions and some information about the value of the stock.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

Let's look at these payoff functions more closely.

When the value of the stock is high, assuming you just reported, then *your prediction* must **exceed** that of the *previous trader's reported prediction* to make **profits**. Why? $(100 - \textit{your prediction})^2$ is smaller than $(100 - \textit{previous trader's reported prediction})^2$ precisely because *your prediction* is a bigger number than the *previous trader's reported prediction.* Therefore, $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2] > 0$. Otherwise, when the value of the stock is high, and *your prediction* is **less than** that of the *previous trader's reported prediction*, you will make **losses**.

When the value of the stock is low, the opposite is true. Assuming you just reported, then *your prediction* must **be less than** that of the *previous trader's reported prediction* to make **profits**. Otherwise, when the value of the stock is low, and *your prediction* **exceeds** that of the *previous trader's reported prediction*, you will make **losses**.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock**

value is revealed to you. Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

Examples

For all the calculations in the examples, assume the following.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction*.

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock value is revealed to you.** Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

1. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 50.00, and yours is 60.00. Is the value of the stock low or high? High What is your payoff for this prediction? 9.00 ECUs

2. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 55.00, and yours is 35.00. Is the value of the stock low or high? High What is your payoff for this prediction? −22.00 ECUs

3. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 67.00, and yours is 72.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −6.95 ECUs

4. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 77.00, and yours is 22.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 54.45 ECUs

Quiz

For the calculations, use the calculator provided in the bottom left portion of this screen. Press the icon and the calculator will become live. To use the scientific calculator, press view and choose the scientific calculator. Provide your numerical answers to two decimal places.

Recall that:

- When the **value of the stock is high**, your payoff is:
  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:
  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

A colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. The virtual urn contains 90 balls: 30 red balls, 30 green balls, and 30 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

We provide the two traders with some **private** information about the draw. This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

1. How many rounds of game play are there? 12.00

2. During the duration of the round, you will be matched with the same individual. Yes

3. In each round, you will be matched with the same individual. No

4. To determine whether there will be another prediction in the round, the computer draws integer 27. Is there going to be another prediction in the round? Yes

5. To determine whether there will be another prediction in the round, the computer draws integer 96. Is there going to be another prediction in the round? No

6. The round payoff is determined at the very end of the round, when the value of the stock is revealed to you. Yes

7. If your round payoff turns out to be negative, then we will zero your round payoff for that round. Yes

8. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 57.00, and yours is 82.00. What is your payoff for this prediction? -34.75 ECUs

9. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 57.00, and yours is 85.00. What is your payoff for this prediction? 16.24 ECUs

10. In the beginning of the round, you, as Trader 1, received private information that the ball is not blue. Suppose that Trader 2's previous prediction was 40.00, and yours is also 40.00. What is your payoff for this prediction? 0.00 ECUs

11. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 40.00, and yours is 50.00. What is your payoff for this prediction? 11.00 ECUs

12. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that your very first prediction is 30.00. What is your payoff for this prediction? −9.00 ECUs [The initial value of 0 was changed to 50 in the other set of treatments, which yields 16.00 ECUs.]

13. Suppose that the color of the ball is revealed to you at the end of the round, and you earned the following payoffs for your predictions within the round: 15.00, −10.00, 25.00, 40.00, 10.00. Recall that in the beginning of each round you are provided with 1,500 ECUs. What is your round payoff? 1,580.00 ECUs

14. Suppose that in the 12 rounds, you earned the following round payoffs: 1,500, 1,000, 2,000, 1,500, 1,000, 1,000, 1,000, 1,000, 1,000, 1,000, 2,000, 2,000. What is your final payoff? 16,000.00 ECUs

15. If you earned 16,000 ECUs in the 12 rounds, your final payoff in euros is what? €8.00

3.2.4 $1 \leq$ Red $\leq$ 30, $20 \leq$ Green $\leq$ 69, $20 \leq$ Blue $\leq$ 69, Red & Green $\rightarrow$ High [AmbStS0]

Recall that the instructions are identical to all participants.

You are about to participate in an experiment about prediction markets. You will spend the next few minutes learning how to make predictions and how your earnings are calculated. All values are denominated in Experimental Currency Units (ECUs). With the completion of this part, your ECUs will be converted into euros at the exchange rate of 2,000 ECUs equals €1.

In each round, you have at your disposal 1,500 ECUs; that is, in each round you can **only** use your starting 1,500 ECUs. There are 12 rounds of game play. Though for the duration of the round you will be paired with the **same** participant, in every new round, you will be matched with a **different** participant. Within a given round, the pair of participants will take on the role of traders that take turns (alternate at) predicting the value of the stock. Specifically, **within a given round**, first, Trader 1 will provide his prediction for the value of the stock, then Trader 2 will provide her prediction for the value of the stock, then Trader 1 will provide his prediction for the value of the stock, then Trader 2, and so on and so forth. Whether or not you are Trader 1 or Trader 2 will be determined by a computer draw **at the beginning of each round**. You can imagine the computer flipping a virtual coin so that the chance of each outcome is equal.

How many predictions within a given round will the two traders report? Again, this is determined by a computer draw. Specifically, after the report of each prediction, the computer will draw an integer from 1 to 100 (all inclusive), where each integer has the same probability of being drawn. If the computer draws an integer below or equal to 95, then there will be one more prediction in the round; otherwise, if the computer draws an integer above 95, then the round ends. Thus, after each prediction, there is 95% chance that there will be one more prediction in the round, and 5% chance that there will be no other prediction and the round will end.

Your prediction for the value of the stock can be any integer from 0 to 100.

To help you decide on your prediction, we will provide next the payoff functions and some information about the value of the stock.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - previous\ trader's\ reported\ prediction)^2 - (100 - your\ prediction)^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(previous\ trader's\ reported\ prediction)^2 - (your\ prediction)^2]$.

Let's look at these payoff functions more closely.

When the value of the stock is high, assuming you just reported, then *your prediction* must **exceed** that of the *previous trader's reported prediction* to make **profits**. Why? $(100 - your\ prediction)^2$ is smaller than $(100 - previous\ trader's\ reported\ prediction)^2$ precisely because *your prediction* is a bigger number than the *previous trader's reported prediction.* Therefore, $0.01[(100 - previous\ trader's\ reported\ prediction)^2 - (100 - your\ prediction)^2] > 0$. Otherwise, when the value of the stock is high, and *your prediction* is **less than** that of the *previous trader's reported prediction*, you will make **losses**.

When the value of the stock is low, the opposite is true. Assuming you just reported, then *your prediction* must **be less than** that of the *previous trader's reported prediction* to make **profits**. Otherwise, when the value of the stock is low, and *your prediction* **exceeds** that of the *previous trader's reported prediction*, you will make **losses**.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock**

value is revealed to you. Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: between 1 and 30 red balls, between 20 and 69 green balls, and between 20 and 69 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

Examples

For all the calculations in the examples, assume the following.

Payoff Functions

The stock has either a **high value** or **low value**.

Your payoff depends on (a) the stock value (high or low), (b) your prediction, and (c) the previous trader's reported prediction.

Specifically:

- When the **value of the stock is high**, your payoff is:

  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:

  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

To make neither losses nor profits (i.e. a payoff of 0), you simply need to replicate the choice of the *previous trader's reported prediction.*

To calculate the payoff of Trader 1's **very first prediction**, we assume that the *previous trader's reported prediction* is 0. [The initial value of 0 was changed to 50 in the other set of treatments.]

The **round payoff** is the summation of all the payoffs of the trader in the round. **Crucially, the round payoff will be determined at the end of the round, when the stock value is revealed to you.** Recall further that in the beginning of the round, you have at your disposal 1,500 ECUs. It is possible that based on the payoffs of your predictions in the round, your funds will go down to zero or even negative. If your **round payoff** is a negative number, then we will zero your round payoff for that round. In the new round, you will be given once again your starting 1,500 ECUs. The **final payoff** is the summation of all the round payoffs of the trader in the 12 rounds played.

In summary, in order to make profits, when the value of the stock is high, *your prediction* must **exceed** that of the *previous trader's reported prediction*, and when the value of the stock is low, *your prediction* must **be less than** that of the *previous trader's reported prediction.*

Information

At the beginning of each round a colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. Furthermore, the color of the ball will **not be revealed to you until the end of the round.**

The virtual urn contains 90 balls: between 1 and 30 red balls, between 20 and 69 green balls, and between 20 and 69 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

Importantly, we will provide the two traders with some **private** information about the draw. **This information is different across the two traders.**

If the drawn ball is red (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

If the drawn ball is green (hence, the value of the stock is high): Trader 1 will be informed that the drawn ball is **not blue**, whereas Trader 2 will be informed that the drawn ball is **green**.

If the drawn ball is blue (hence, the value of the stock is low): Trader 1 will be informed that the drawn ball is **blue**, whereas Trader 2 will be informed that the drawn ball is **not green**.

This information is presented in a tabular form.

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

Recall that the color of the ball (hence, whether the stock value is low or high) will be revealed to you at the end of the round. The round payoff will then be determined.

1. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 50.00, and yours is 60.00. Is the value of the stock low or high? High What is your payoff for this prediction? 9.00 ECUs

2. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 55.00, and yours is 35.00. Is the value of the stock low or high? High What is your payoff for this prediction? −22.00 ECUs

3. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 67.00, and yours is 72.00. Is the value of the stock low or high? Low What is your payoff for this prediction? −6.95 ECUs

4. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 77.00, and yours is 22.00. Is the value of the stock low or high? Low What is your payoff for this prediction? 54.45 ECUs

Quiz

For the calculations, use the calculator provided in the bottom left portion of this screen. Press the icon and the calculator will become live. To use the scientific calculator, press view and choose the scientific calculator. Provide your numerical answers to two decimal places.

Recall that:

- When the **value of the stock is high**, your payoff is:
  $0.01[(100 - \textit{previous trader's reported prediction})^2 - (100 - \textit{your prediction})^2]$.

- When the **value of the stock is low**, your payoff is:
  $0.01[(\textit{previous trader's reported prediction})^2 - (\textit{your prediction})^2]$.

A colored ball (red, green, or blue) is drawn by the computer from a virtual urn. The color of the drawn ball will determine the value of the stock; that is, whether the stock has a **high value** or **low value**. The virtual urn contains 90 balls: between 1 and 30 red balls, between 20 and 69 green balls, and between 20 and 69 blue balls.

- If the drawn ball is red, then the stock value is high.
- If the drawn ball is green, then the stock value is again high.
- If the drawn ball is blue, then the stock value is low.

We provide the two traders with some **private** information about the draw. This information is presented in a tabular form.

1. How many rounds of game play are there? 12.00

2. During the duration of the round, you will be matched with the same individual. Yes

	Private Information	
Ball Drawn	**Trader 1**	**Trader 2**
Red	Not Blue	Not Green
Green	Not Blue	Green
Blue	Blue	Not Green

3. In each round, you will be matched with the same individual. No

4. To determine whether there will be another prediction in the round, the computer draws integer 27. Is there going to be another prediction in the round? Yes

5. To determine whether there will be another prediction in the round, the computer draws integer 96. Is there going to be another prediction in the round? No

6. The round payoff is determined at the very end of the round, when the value of the stock is revealed to you. Yes

7. If your round payoff turns out to be negative, then we will zero your round payoff for that round. Yes

8. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that Trader 2's previous prediction was 57.00, and yours is 82.00. What is your payoff for this prediction? −34.75 ECUs

9. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 57.00, and yours is 85.00. What is your payoff for this prediction? 16.24 ECUs

10. In the beginning of the round, you, as Trader 1, received private information that the ball is not blue. Suppose that Trader 2's previous prediction was 40.00, and yours is also 40.00. What is your payoff for this prediction? 0.00 ECUs

11. In the beginning of the round, you, as Trader 2, received private information that the ball is green. Suppose that Trader 1's previous prediction was 40.00, and yours is 50.00. What is your payoff for this prediction? 11.00 ECUs

12. In the beginning of the round, you, as Trader 1, received private information that the ball is blue. Suppose that your very first prediction is 30.00. What is your payoff for this prediction? −9.00 ECUs [The initial value of 0 was changed to 50 in the other set of treatments, which yields 16.00 ECUs.]

13. Suppose that the color of the ball is revealed to you at the end of the round, and you earned the following payoffs for your predictions within the round: 15.00, −10.00, 25.00, 40.00, 10.00. Recall that in the beginning of each round you are provided with 1,500 ECUs. What is your round payoff? 1,580.00 ECUs

14. Suppose that in the 12 rounds, you earned the following round payoffs: 1,500, 1,000, 2,000, 1,500, 1,000, 1,000, 1,000, 1,000, 1,000, 1,000, 2,000, 2,000. What is your final payoff? 16,000.00 ECUs

15. If you earned 16,000 ECUs in the 12 rounds, your final payoff in euros is what? €8.00

3.3 Part 3

In this part of the study, you will complete a questionnaire. The questionnaire asks you to answer some questions about yourself. Please note that your individual data will be kept strictly confidential.

1. What is your age?

2. What is your gender?

Male
Female

3. What is your degree in?

References

Ales, Laurence, and Christopher Sleet. “Revision Proofness.” *Journal of Economic Theory* 152: (2014) 324–55.

Aliprantis, Charalambos, and Kim Border. *Infinite Dimensional Analysis: A Hitchhiker’s Guide*. Springer, 2013.

Asheim, Geir B. “Individual and Collective Time-Consistency.” *The Review of Economic Studies* 64, 3: (1997) 427–43.

Cerreia-Vioglio, Simone, Fabio Maccheroni, Massimo Marinacci, and Luigi Montrucchio. “Uncertainty Averse Preferences.” *Journal of Economic Theory* 146, 4: (2011) 1275–1330.

Chen, Yiling, Stanko Dimitrov, Rahul Sami, Daniel M Reeves, David M. Pennock, Robin D. Hanson, Lance Fortnow, and Rica Gonen. “Gaming Prediction Markets: Equilibrium Strategies with a Market Maker.” *Algorithmica* 58, 4: (2010) 930–69.

Dimitrov, Stanko, and Rahul Sami. “Non-Myopic Strategies in Prediction Markets.” In *Proceedings of the 9th ACM Conference on Electronic Commerce*. 2008, 200–9.

Fudenberg, Drew, and David Levine. “Subgame-Perfect Equilibria of Finite-and Infinite-Horizon Games.” *Journal of Economic Theory* 31, 2: (1983) 251–68.

Fudenberg, Drew, and David K. Levine. “Limit Games and Limit Equilibria.” *Journal of Economic Theory* 38, 2: (1986) 261–79.

Hansen, Lars Peter, and Thomas J. Sargent. “Robust Control and Model Uncertainty.” *American Economic Review: Papers and Proceedings* 91, 2: (2001) 60–6.

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji. “A Smooth Model of Decision Making Under Ambiguity.” *Econometrica* 73, 6: (2005) 1849–92.

Kyle, Albert S. “Continuous Auctions and Insider Trading.” *Econometrica* 53, 6: (1985) 1315–35.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini. “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences.” *Econometrica* 74, 6: (2006a) 1447–98.

———. “Dynamic Variational Preferences.” *Journal of Economic Theory* 128, 1: (2006b) 4–44.

Markowitz, Harry M. “Portfolio Selection.” *Journal of Finance* 7: (1952) 77–91.

Ostrovsky, Michael. “Information Aggregation in Dynamic Markets With Strategic Traders.” *Econometrica* 80, 6: (2012) 2595–647.

Schlag, Karl, and Andriy Zapechelnyuk. “Compromise, don’t optimize: Generalizing perfect Bayesian equilibrium to games with ambiguity.” *Unpublished manuscript, University of Vienna and University of St. Andrews* .

Tobin, James. “Liquidity Preference as Behavior Toward Risk.” *The Review of Economic Studies* 25: (1958) 65–86.

Tuy, Hoang. “A New Topological Minimax Theorem with Application.” *Journal of Global Optimization* 50, 3.